LECTURE NOTES

ON

THEORY OF MACHINES

SAMBIT KUMAR SAHOO

LECTURE IN AUTOMOBILE ENGG.

GOVT. POLYTECHNIC, BOUDH

SYLLABUS

1.0 Simple mechanism

1.1 Link, kinematic chain, mechanism, machine

1.2 Inversion, four bar link mechanism and its inversion

1.3 Lower pair and higher pair

01.4 Cam and followers

2.0 Friction

2.1 Revision of topic previously taught

2.2 Friction between nut and screw for square thread, screw jack

2.3 Bearing and its classification, Description of roller, needle roller & ball bearings.

2.4 Torque transmission in flat pivot & conical pivot bearings.

2.5 Flat collar bearing of single and multiple types.

2.6 Torque transmission for single and multiple clutches

2.7 Working of simple frictional brakes.

2.8 Working of Absorption type of dynamometer

3.0 Power Transmission

3.1 Concept of power transmission

3.2 Type of drives, belt, gear and chain drive.

3.3 Computation of velocity ratio, length of belts (open&cross) with and without slip.

3.4 Ratio of belt tensions, centrifugal tension and initial tension.

3.5 Power transmitted by the belt

. 3.6 V-belts and V-belts pulleys.

3.7 Concept of crowning of pulleys.

3.8 Gear drives and its terminology.

3.9 Gear trains, Working principle of simple, compound, reverted and epicyclic gear trains.

4.0 Governors and Flywheel

4.1 Function of governor

4.2 Classification of governor

4.3 Working of Watt, Porter, Proel and Hartnel l governors.

4.4 Conceptual explanation of sensitivity, stability and isochronism .

4.5 Function of flywheel.

4.6 Comparison between flywheel & governor.

4.7 Fluctuation of energy and coefficient of fluctuation of speed.

5.0 Balancing of Machine

5.1 Concept of static and dynamic balancing.

5.2 Static balancing of rotating parts.

5.3 Principles of balancing of reciprocating parts.

5.4 Causes and effect of unbalance.

5.5 Difference between static and dynamic balancing

6.0 Vibration of machine parts

6.1 Introduction to Vibration and related terms (Amplitude, time period and frequency, cycle)

6.2 Classification of vibration.

6.3 Basic concept of natural, forced & damped vibration

6.4 Torsional and Longitudinal vibration. 6.6 Causes & remedies of vibration.

Learning Resources:

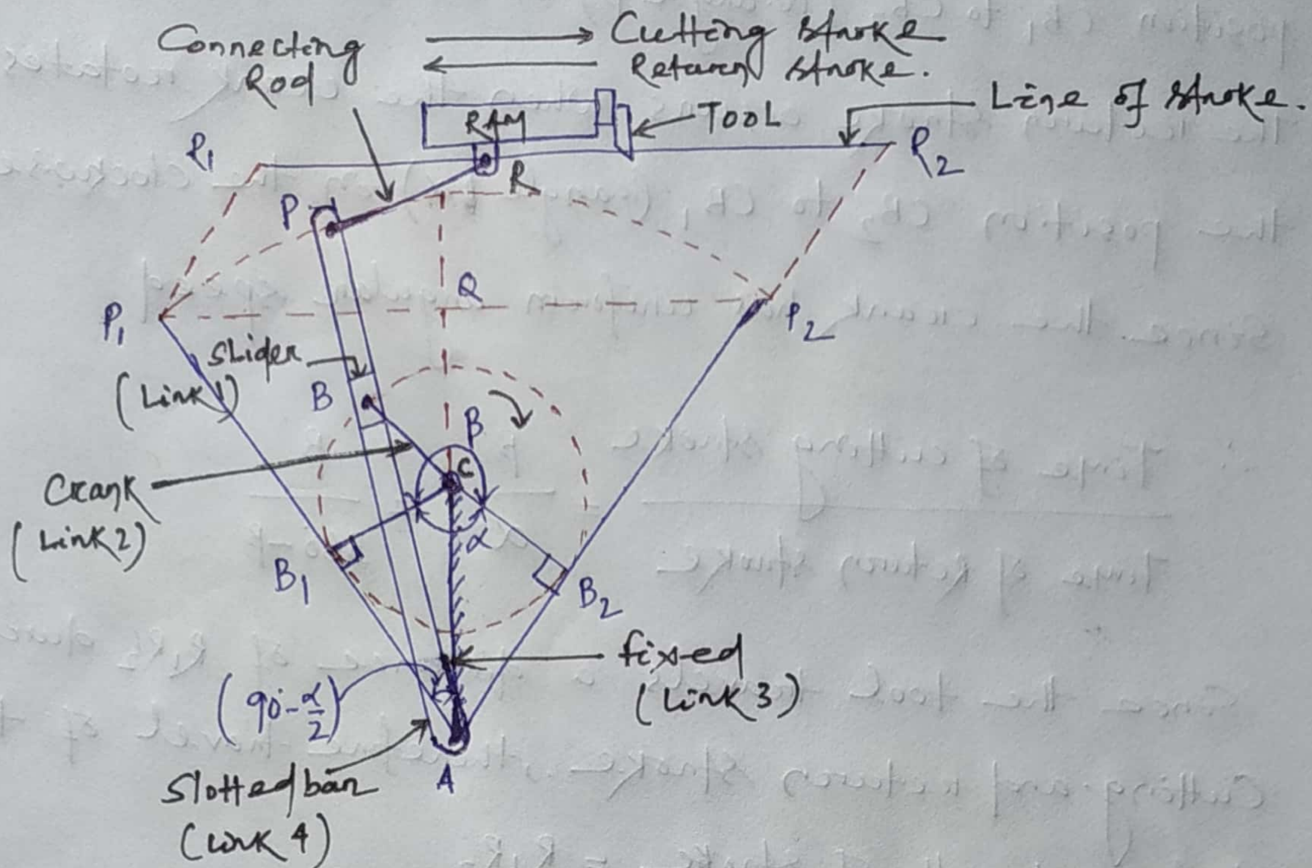
Text Books Theory of Machines by R S Khrmi

Theory of Machines by R K Rajput

Theory of Machines by S R Rattan

Reference Book

Theory of Machines by P L Ballaney



In this mechanism, the link AC (i.e. link 3) forming turning pair is fixed. the link 3 corresponds to the connecting rod of a reciprocating steam engine. the driving crank CB revolves with uniform angular speed about a fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP & thus causes AP to oscillate about the pivoted point A. the short link PR the transmit the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram is perpendicular to AC.

In the extreme position, AP_1 & AP_2 are tangential to the circle & the cutting tool is at the end of the stroke. The forward or Cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (i.e. angle β) in the clockwise direction.

The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (i.e. angle α) in the clockwise direction.

Since the crank has uniform angular speed

$$\therefore \frac{\text{Time of cutting stroke}}{\text{Time of Return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta}$$

Since the tool travels a distance of R_1R_2 during Cutting and return stroke, therefore travel of the tool or length of stroke $= R_1R_2$

$$= P_1P_2$$

$$= 2P_1Q$$

$$= 2AP_1 \sin \angle P_1AQ$$

$$= 2AP_1 \sin\left(90^\circ - \frac{\alpha}{2}\right) \quad (\because AP_1 = AP)$$

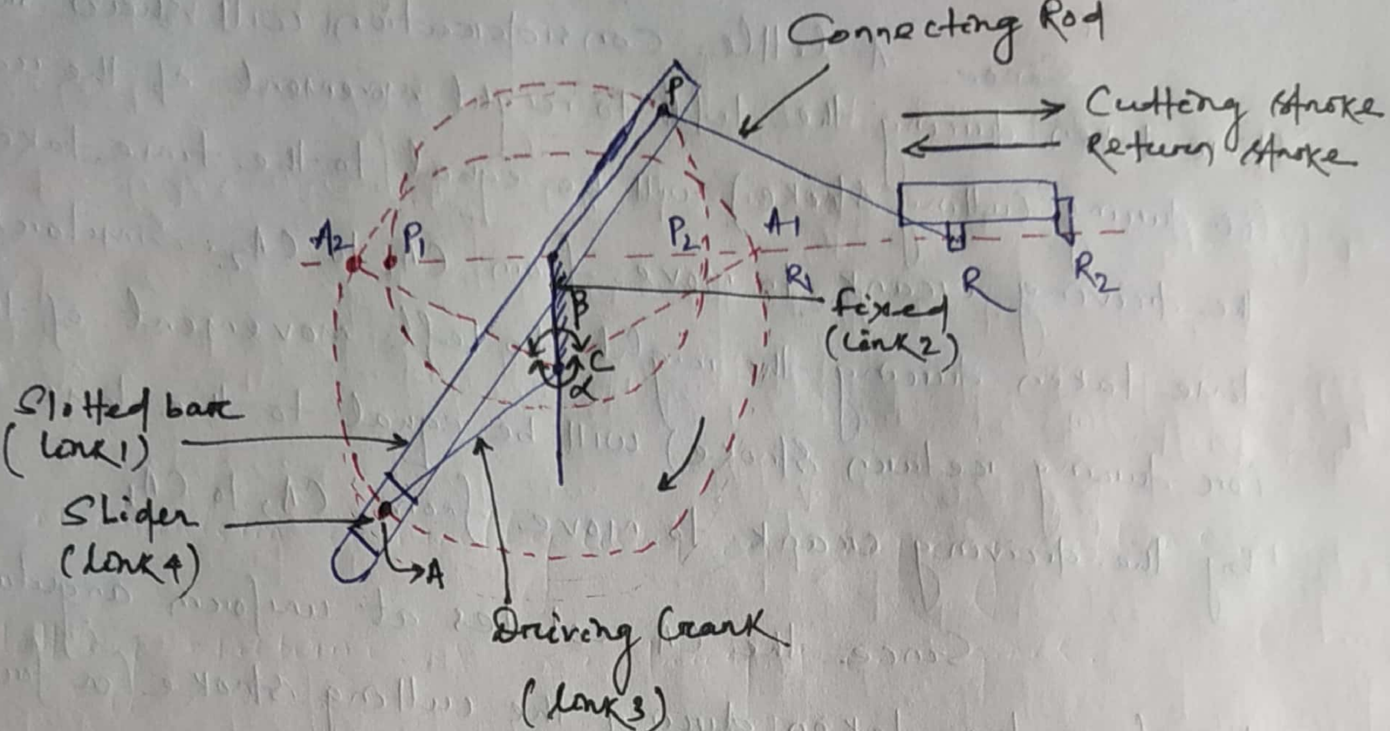
$$= 2AP \cos\frac{\alpha}{2}$$

$$= 2AP \times \frac{CB_1}{AC} \quad \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

$$= 2AP \times \frac{CB}{AC} \quad (\because CB_1 = CB)$$

→ This mechanism is mostly used in shaping machine, slotting machine, & in rotary I.C. engines.

NOTE Since we see angle β is greater than α, the crank rotates with uniform angular speed, so the return stroke is completed within shorter time. This is called Quick return motion mechanism.



In this mechanisim the link CD (i.e. link 2) forming the turning pair is fixed. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. the slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting Rod PR carries the ram at R to which the cutting tool is fixed. the motion of the tool is constrained along the line RD produced. i.e. along a line passing through D & perpendicular to CD.

When the driving crank CA moves from the position CA_1 to CA_2 (or the link DP from the position DP_1 to DP_2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a Distance 2PD.

1101172

Scanned with CamScanner

(4) Now when the driving crank moves from the position (4)
CA_2 to CA_1 (or the link QP from QP_2 to QP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

~~Since~~ A little consideration will show that the time taken during the left to right movement of the ram (i.e. during cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly, the time taken during the right to left movement of the ram (or during return stroke) will be equal to the time taken by the driving crank to move from CA_2 to CA_1.

Since the CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words the mean speed of the ram during the cutting stroke is less than the mean speed during the return stroke.

The ratio betn. the time taken during the cutting & return stroke is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \text{ or } \frac{360^\circ - \beta}{\beta}$$

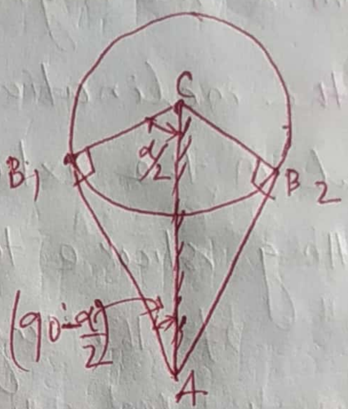
Note In order to find the length of effective stroke R_1R_2 mark $P_1R_1 = P_2R_2 = PR$.
. the length of the effective stroke is equal to $2PQ$.

→ It is mostly used in shaping & slotting machine.

Scanned with CamScanner

(5)

Q) A crank and slotted lever mechanism used in a shaper has a centre distance of 300mm bet^n the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120mm. find the ratio of the time of return stroke to the time of cutting.



Sol^n Given data

$AC = 300mm.$

$CB_1 = 120mm.$

we know in the ΔCAB_1,

$\therefore \sin \angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right)$

$\Rightarrow \sin \angle CAB_1 = \frac{CB_1}{AC} = \frac{120}{300} = 0.4$

$\cancel{\angle CAB_1 = 90° - \frac{\alpha}{2}}$

$\Rightarrow \angle CAB_1 = \sin^{-1}(0.4) = 23.6°$

$\therefore \sin \angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right)$

$\Rightarrow \angle CAB_1 = 90° - \frac{\alpha}{2}$

$\Rightarrow 23.6° = 90° - \frac{\alpha}{2}$

$\Rightarrow \frac{\alpha}{2} = 90° - 23.6° = 66.4°$

$\Rightarrow \alpha = 2 \times 66.4° = 132.8°$

1101171

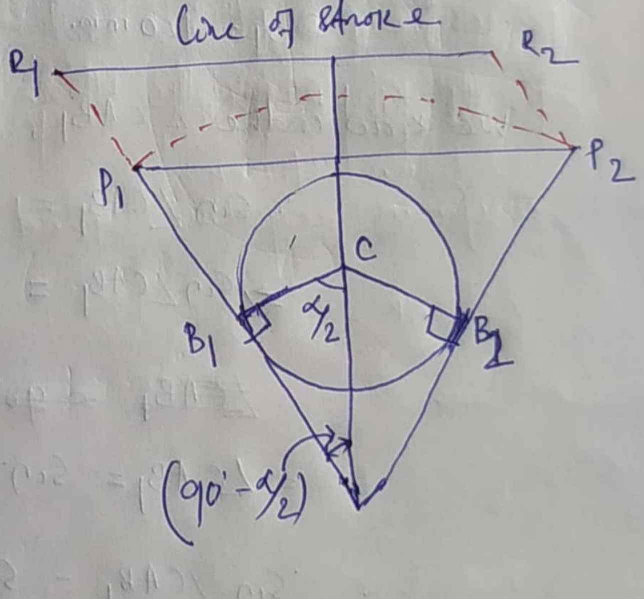
Scanned with CamScanner

(6) We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of Return stroke}} = \frac{360^\circ - \alpha}{\alpha} = \frac{360^\circ - 132.8^\circ}{132.8^\circ}$$

$$= 1.72 \text{ Ans.}$$

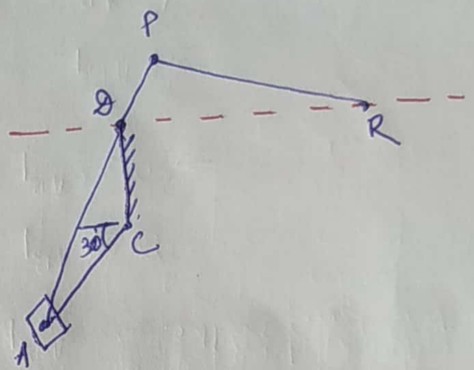
Q.7 In a crank & slotted lever quick return motion mechanisim, the distance between the fixed centres is 240 mm. & the length of the driving crank is 120mm. find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm find the length of the stroke, if the line of stroke passes through the extreme position of the free end of the lever.



Scanned with CamScanner

(8)

Q) In a withworth quick return motion mechanism as shown in figure The distance bet^n the fixed centers is 50 mm and the length of the driving crank is 75 mm. The length of the slotted lever is 150 mm & the length of the connecting rod is 135 mm. find the ratio of the time of cutting stroke ~~and also the effective~~ to the time of return stroke. ~~stroke~~.



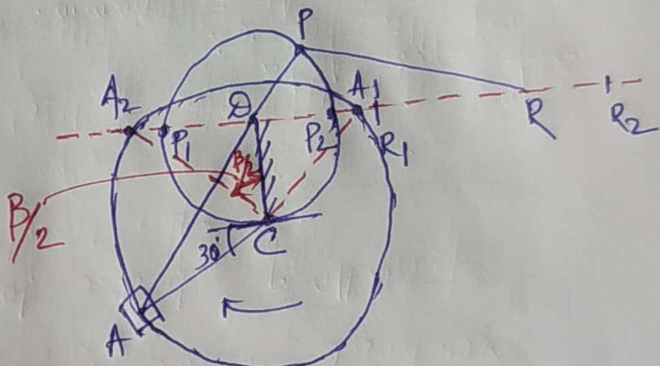
Given data

fixed center = $CQ = 50\,mm$.

Length of driving crank = $CA = 75\,mm$.

length of slotted lever = $PA = 150\,mm$.

length of connecting rod = $PR = 135\,mm$.



Extreme position of driving crank as shown in the figure.

then $\cos \beta/2 = \frac{CQ}{CA_2} = \frac{50}{75}$

$= 0.667 \quad (\because CA_2 = CA)$

$\Rightarrow \beta/2 = \cos^{-1}(0.667) = 48.2°$

$\Rightarrow \underline{\beta = 96.4°}$

$\therefore$ Ratio of the time of Cutting stroke to time of Return stroke

$$\Rightarrow \frac{\text{Time of cutting stroke}}{\text{Time of Return stroke}} = \frac{360-\beta}{\beta} = \frac{360-96.4}{96.4}$$

$= 2.735$

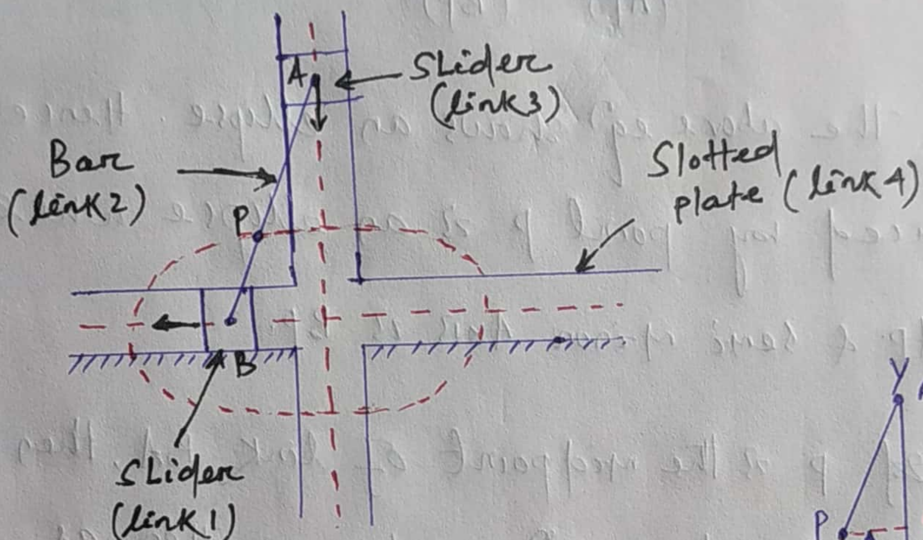
~~length of effective stroke~~

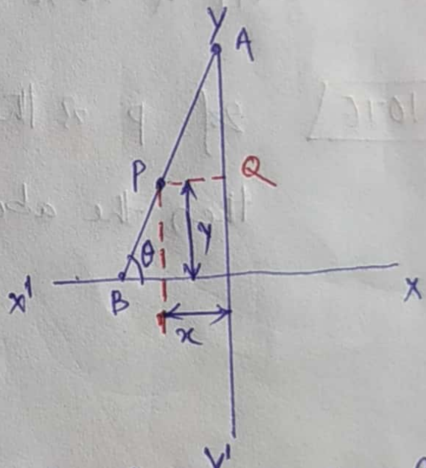
(9) Double slider crank chain.

A Kinematic chain which consists of two turning pairs & two sliding pairs is known as double slider crank chain.

Inversion of Double slider crank chain.

(1) Elliptical trammels.





This is obtained by fixing the slotted plate (link 4). The link 4 has two straight grooves cut in it at right angles to each other. The link 1 and link 3 are known as slider & form sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pairs with link 1 & link 3.

When the link 1 & 3 slide along their respective grooves any point on the link 2 such as P traces out an ellipse on the surface of link 4. It shows that AP & BP are the semi-major axis & semi-minor axis of the ellipse respectively.

(10) Let us take OX & OY as horizontal & vertical axes and let the link BA is inclined at an angle θ with the horizontal. the coordinates of the point P on the link BA will be

$$x = PQ = AP\cos\theta \Rightarrow \frac{x}{AP} = \cos\theta \quad —①$$

$$y = PR = BP\sin\theta \Rightarrow \frac{y}{BP} = \sin\theta \quad —②$$

∴ squaring & adding in eqⁿ ① & ②

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

The above eqⁿ shows an ellipse. Hence the path traced by point P is an ellipse whose semi-major axis is AP & semi minor Axis is BP.

NOTE If P is the mid point of link BA, then AP = BP.
then the above eqⁿ can be written as

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1.$$

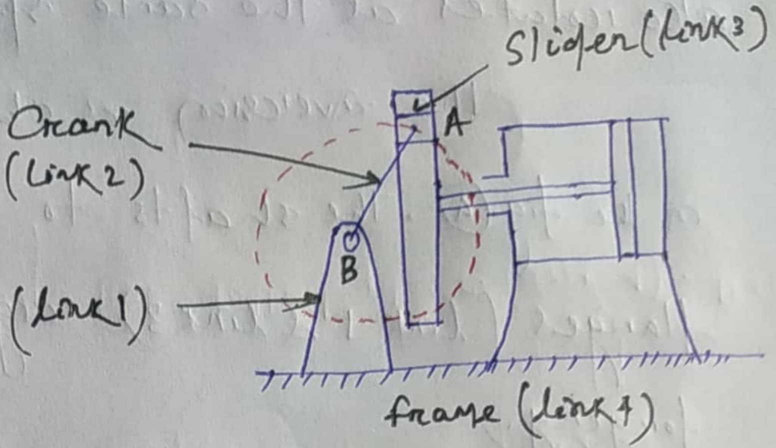
$$\text{or} \quad x^2 + y^2 = AP^2$$

This is the eqⁿ of a circle whose radius is AP.

Hence if P is the mid-point of link BA, it will trace a circle.

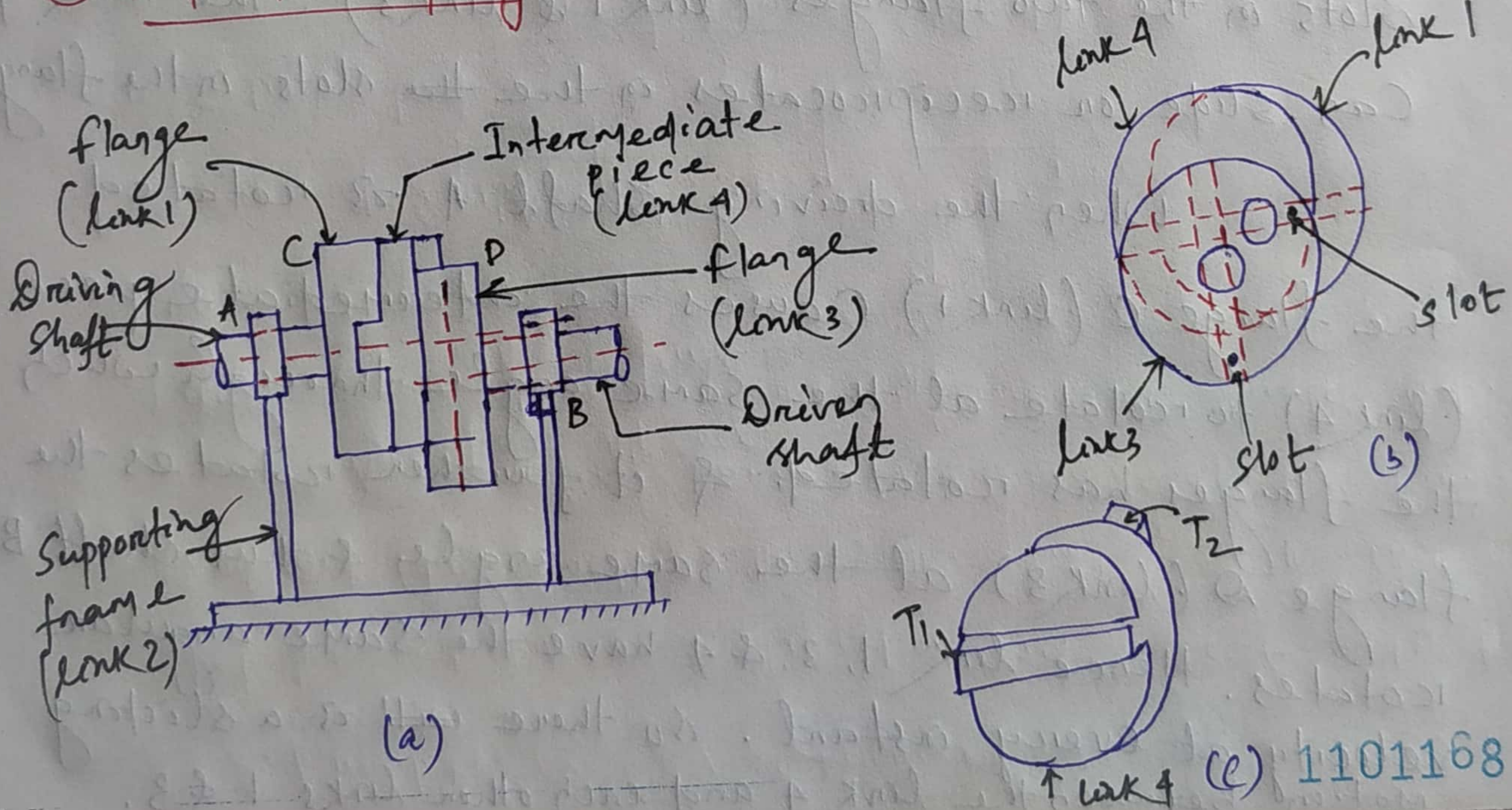
→ This is used for drawing ellipse.

② Scotch yoke Mechanism :



This mechanism is used for converting rotary motion into a reciprocating motion. the inversion is obtained by fixing either the link 1 or link 3. In this mechanism when the link 2 rotates about B as centre, the link 4 reciprocates. the fixed link 1 guides the frame.

③ Oldham's Coupling :-



1101168

(12) It is used for connecting two parallel shafts whose axes are at a small distance apart. the shaft are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed.

the inversion is obtained by fixing the link 2. on the fig (a). The shafts to be connected have two flanges (link 1 & link 3) rigidly fastened at their end by forging.

The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut on their inner faces, as shown in fig (b). The intermediate piece (link 4) which is a circular disc, have two tongues, (i.e. diametrical projections) T_1 & T_2 on each face at ~~ring~~ right angles to each other, as shown on figure (c) the toungues on the link 4 closely fit onto the slots on the two flanges (link 1 & link 3). the link 4 Can slide on reciprocates on the ~~the~~ slots on the flange.

When the driving shaft A is rotated the flange C (link 1) Causes the intermediate piece (link 4) to rotate at the same angle through which the flanges has rotated. & it further rotates the flange D (link 3) at the same angle & thus shaft B rotates. Hence link 1, 3, & 4 have the same angular velocity at every instant. So there ~~will~~ is a sliding motion between the link 4 and each other links 1 & 3.

(13) If the distance between the axis of the shaft is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shaft. So the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

let ω = Angular velocity of each shaft in rad/s.

r = Distance betn the axes of the shafts in metres.

$\therefore$ Maximum sliding speed of each tongue (in m/s)

$$\boxed{v = r\omega}$$

Q) The distance betn two parallel shaft is 18 mm and they are connected by oldham's coupling. The driving shaft revolves at 160 rpm. What will be the maximum speed of sliding of the tongue of the intermediate piece along its groove?

Soln $\omega = \frac{2\pi N}{60}$

1101167

Bearing

Bearing is a mechanical element that permits relative motion between two parts, such as the shaft and the housing with minimum friction.

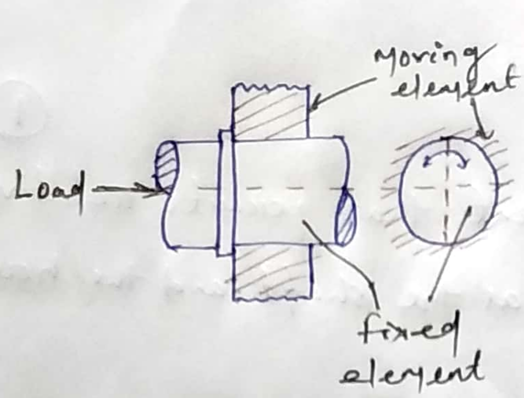
fun.: → the bearing ensures free rotation of the shaft on the axle with minimum friction.

→ It supports the shaft or axle & holds in correct position.

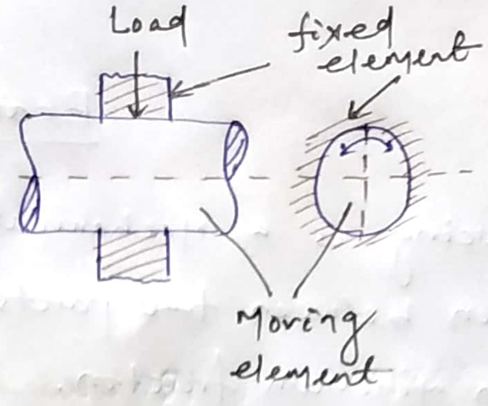
→ It also takes up the forces that act on the shaft or the axle & transmit them to the frame or the foundation

Classification of Bearing

- Bearing
  - Sliding contact bearing
    - Sleeve bearing
      - w.r.t. dir. of Load
        - Axial Load or Thrust bearing
          - pivot or foot step Bearing
            - Flat pivot
            - Conical pivot
          - collar Bearing
            - flat collar
            - Conical collar
              - Single collar
              - Multi collar
        - Radial Load or Journal bearing
          - full J.B.
          - partial J.B.
          - fitted J.B.
      - w.r.t. type of Lubrication
        - Thin film lubrication
        - thick film
          - Hydrodynamic Lubrication
          - Hydrostatic Lubrication.
        - zero film
      - w.r.t. type of bushes
        - Solid bush
        - Split bush
    - Slipper or guide bearing
  - (Anti friction bearing) Roller contact Bearing
    - Ball bearing
      - Deep groove
      - Angular Contact
      - Self aligning
      - Thrust
    - Roller bearing
      - Cylindrical
      - Spherical
      - Taper
      - Needle



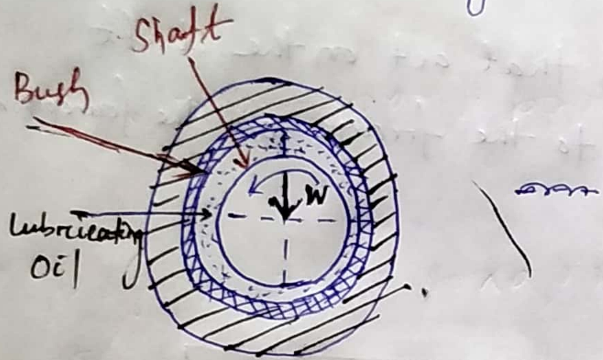
Radial bearing



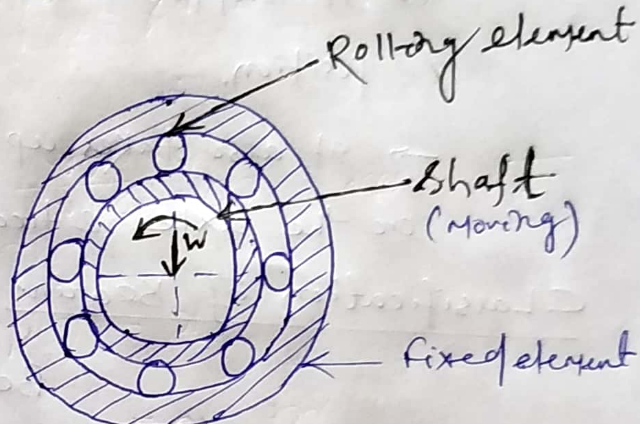
Thrust bearing

In radial bearing the load acts perpendicular to the dirⁿ of motion of the moving element.

In thrust bearing the load acts along the axis of rotation.

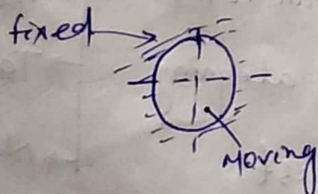


Sliding Contact bearing

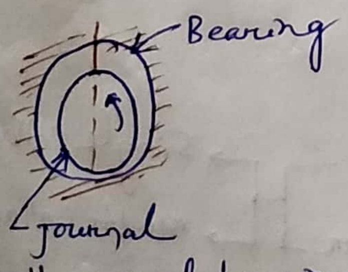


Rolling Contact Bearing

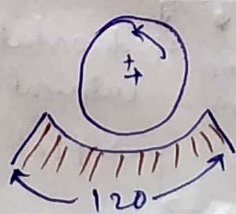
→ The sliding ~~contact~~ takes place along the surfaces of contact betⁿ the moving element & the fixed element.



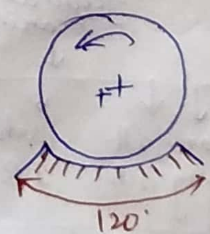
→ In R.C.B., the ~~steel~~ rollers are interposed betⁿ the moving & fixed element. The balls offer rolling friction at two points for each ball or roller.



+) full journal bearing



partial journal Bearing



fitted journal Bearing

① Full J.B. :- when the angle of contact of the bearing with the journal is 360°, then the Bearing is called F.J.B.

② partial J.B. :- When the angle of contact of the bearing is 120°

③ fitted J.B. When a partial journal bearing has no clearance i.e. the diameters of the journal & bearing are equal, then the Bearing is called fitted Bearing.

① Thick film lubrication :- Metal to metal contact is always avoided (i.e. no metal to metal contact)

② Thin film L. :- Metal to metal contact will always present at any speed.

→ the lubricant is ~~always~~ used to reduce surface co-efficient of friction only.

③ Zero film L. → When metal behaves like lubricant.

eg C.I, graphite.

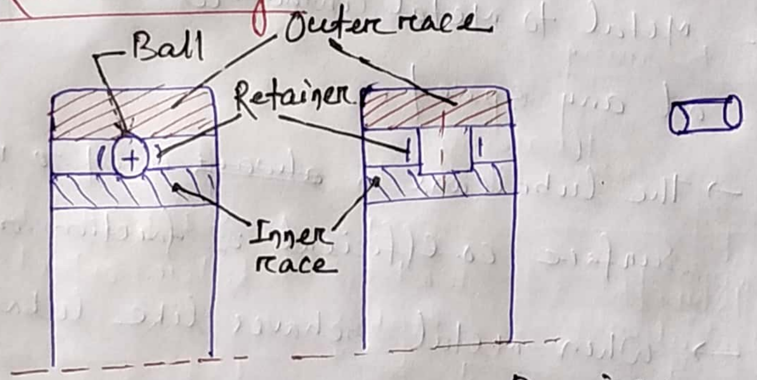
Rolling contact bearing :-

In rolling contact bearing, the contact betn the bearing surface is rolling instead of sliding as in sliding contact bearings.

→ But we have already know that ordinary sliding bearing starts from rest with practically metal to metal contact & has a high coefficient of friction. So in rolling contact bearing have low starting friction than sliding contact bearing.

Due to this low friction offered by rolling contact bearings. these are called antifriction bearing.

Ball & Roller Bearing



Ball Bearing   Roller Bearing

The ball & roller bearings consists of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing. In between the inner and outer race there are balls & rollers. A number of balls or rollers are used and these are held at a proper distance by retainers so that they do not touch each other. the retainers are thin strip & is usually in two parts ~~are~~ which are assembled after the balls have been properly spaced.

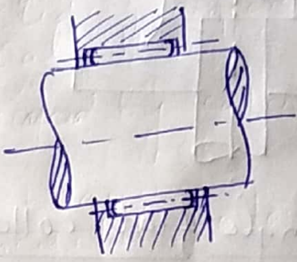
The ball bearings are used for light loads & Roller bearings are used for heavier loads.

Needle roller bearing :-

Needle bearings are characterised by cylindrical rollers of very small diameter and relatively long length. They are also called as 'quill' bearings.

→ the length to diameter ratio of needles is more than four.

→ Needle bearings are used with or without inner & outer races.



→ It is suited for applications involving oscillatory motion such as piston pin bearings, rocker arms and universal joints.

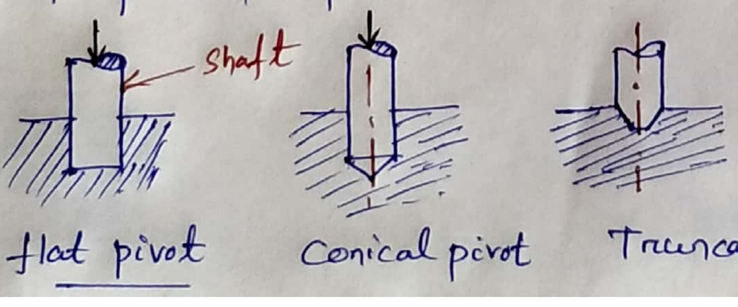
Pivots & Collars :-

When a rotating shaft is subjected to an axial load, the thrust (axial force) is taken either by a pivot or a collar.

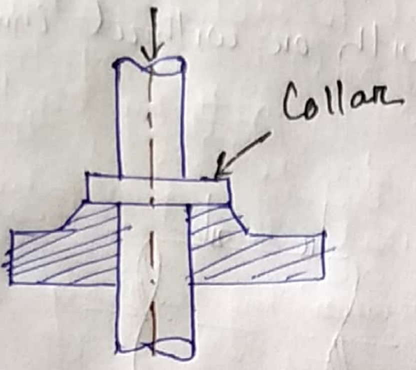
ex :- shafts of a steam turbine & propeller shaft of a ship.

pivot The bearing surface placed at the end of a shaft to take the axial thrust are known as pivot.

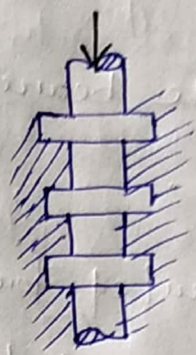
→ The pivot may have a flat surface or conical surface. When the cone is truncated, it is then known as truncated or trapezoidal pivot.



Collar bearing: A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface, the surface of the collar may be plane (flat) normal to the shaft or of a conical shape.

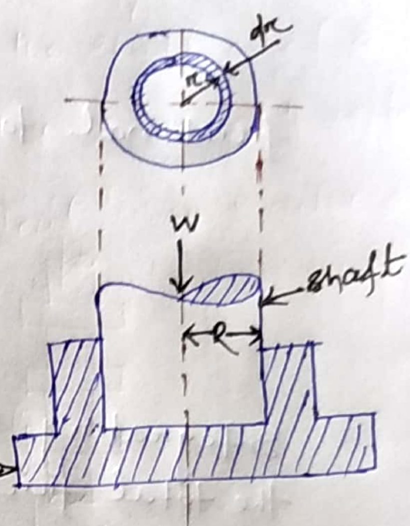


Single flat collar



Multiple flat collar

When a vertical shaft rotates in a flat pivot bearing, the sliding friction will be ~~the~~ along the surface of contact betⁿ. the shaft and the bearing.



flat pivot bearing

Let W = Load transmitted over the bearing surface

R = Radius of bearing surface

p = Intensity of pressure per unit area of bearing surface betⁿ rubbing surface

μ = Coefficient of friction

We will consider the following two cases.

① Considering uniform pressure :

When the pressure is uniformly distributed over the bearing area, then

$$\text{pressure} = \frac{\text{axial force}}{\text{cross-sectional area}} = \frac{W}{\pi R^2}$$

Consider a ring of radius 'r' & thickness 'dr' of the bearing area.

$\therefore$ Area of bearing surface, $A = 2\pi r . dr$

$\therefore$ Load transmitted to the ring, $\delta W = p \times A$
$= p \times 2\pi r . dr$ —— ①

1101163

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu \cdot \delta W$$
$$= \mu \cdot p \times 2\pi r \cdot dr$$
$$= 2\pi\mu \cdot p \cdot r \cdot dr$$

frictional torque on the ring,

$$T_r = F_r \times r$$
$$= 2\pi\mu \cdot p \cdot r \cdot dr \times r$$
$$= 2\pi\mu p r^2 dr \quad \text{——— (2)}$$

Integrating this eqn. within the limits from 0 to R, the total frictional torque on the pivot bearing

$$T = \int_0^R 2\pi\mu p r^2 dr$$
$$= 2\pi\mu p \int_0^R r^2 dr$$
$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_0^R$$
$$= 2\pi\mu p \times \frac{R^3}{3}$$
$$= \frac{2}{3}\pi\mu p R^3$$
$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 \qquad \left(\because p = \frac{W}{\pi R^2}\right)$$
$$\Rightarrow T = \frac{2}{3}\mu W R$$

When the shaft rotates at ω rad/s., then the power lost in the friction,

$$P = T \cdot \omega$$
$$\Rightarrow P = T \times \frac{2\pi N}{60} \qquad \left(\because \omega = \frac{2\pi N}{60}\right)$$

Where N = speed of shaft in R.P.M.

② Considering uniform wear :-

We know that the rate of wear depends upon the intensity of pressure (P) & the velocity of rubbing surface (V). It is assumed that the rate of wear is proportional to the product of intensity of pressure & the velocity of rubbing surface (i.e. P×V).

Since the velocity of rubbing surface increases with the distance (i.e. radius r) from the axis of the bearing therefore for uniform wear.

$$p \cdot r = C \text{ (constant)}$$

$$\Rightarrow p = \frac{C}{r}$$

Load transmitted to the ring

$$\delta W = p \times 2\pi r \cdot dr$$

$$= \frac{C}{r} \times 2\pi r \cdot dr$$

∴ total load transmitted to the bearing

$$W = \int_0^R 2\pi C \cdot dr.$$

$$= 2\pi C [r]_0^R$$

$$\Rightarrow W = 2\pi C R$$

$$\Rightarrow C = \frac{W}{2\pi R}$$

∴ we known that frictional torque acting on the ring

$$T_r = 2\pi \mu p r^2 dr$$

$$= 2\pi \mu \times \frac{C}{r} \times r^2 dr \qquad \left(\because p = \frac{C}{r}\right)$$

$$\Rightarrow T_r = 2\pi \mu \cdot C \cdot r \cdot dr$$

$\therefore$ Total frictional torque on the bearing.

$$T = \int_0^R 2\pi \mu \cdot C \cdot r \cdot dr$$

$$= 2\pi \mu C \left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi \mu C \times \frac{R^2}{2}$$

$$= \pi \mu C R^2$$

$$= \pi \mu \times \frac{W}{2\pi R} \times R^2 \qquad \left(\because C = \frac{W}{2\pi R}\right)$$

$$\Rightarrow T = \frac{1}{2} \times \mu W R$$

H.W

Q. A vertical shaft 150 mm in diameter rotating at 100 rpm rests on a flat end foot-step bearing. The shaft carries a vertical load of 20KN. Assuming uniform pressure distribution & coefficient of friction equal to 0.05, estimate power lost on friction.

Friction clutches.

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. the axes of the two shafts are coincident.

→ Its application is found in cases in which power is to be delivered to machine partially or fully loaded. The force of friction is used to start the driven shaft from rest & gradually brings it up to the proper speed without excessive slipping of the friction surface

→ In Automobile, friction clutches is used to connect the engine to the driven shaft. During clutch operation care should be taken & friction surfaces engage easily & gradually brings the driven shaft up to proper speed.

→ There are ~~three~~ friction clutches

① Disc / plate clutches (Single plate or multiple plate clutch)

② Cone clutches.

③ Centrifugal clutches.

1) Single plate clutch.

A single plate clutch, consists of a clutch plate whose both side are faced with a friction material. It is mounted on the hub which is free to move axially along the splines of the driven shaft the pressure plate is mounted inside the clutch body, which is bolted to the flywheel. Both the pressure plate & the flywheel rotate with the engine crankshaft.

the pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The release levers are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft & moves forward when the clutch pedal is pressed.

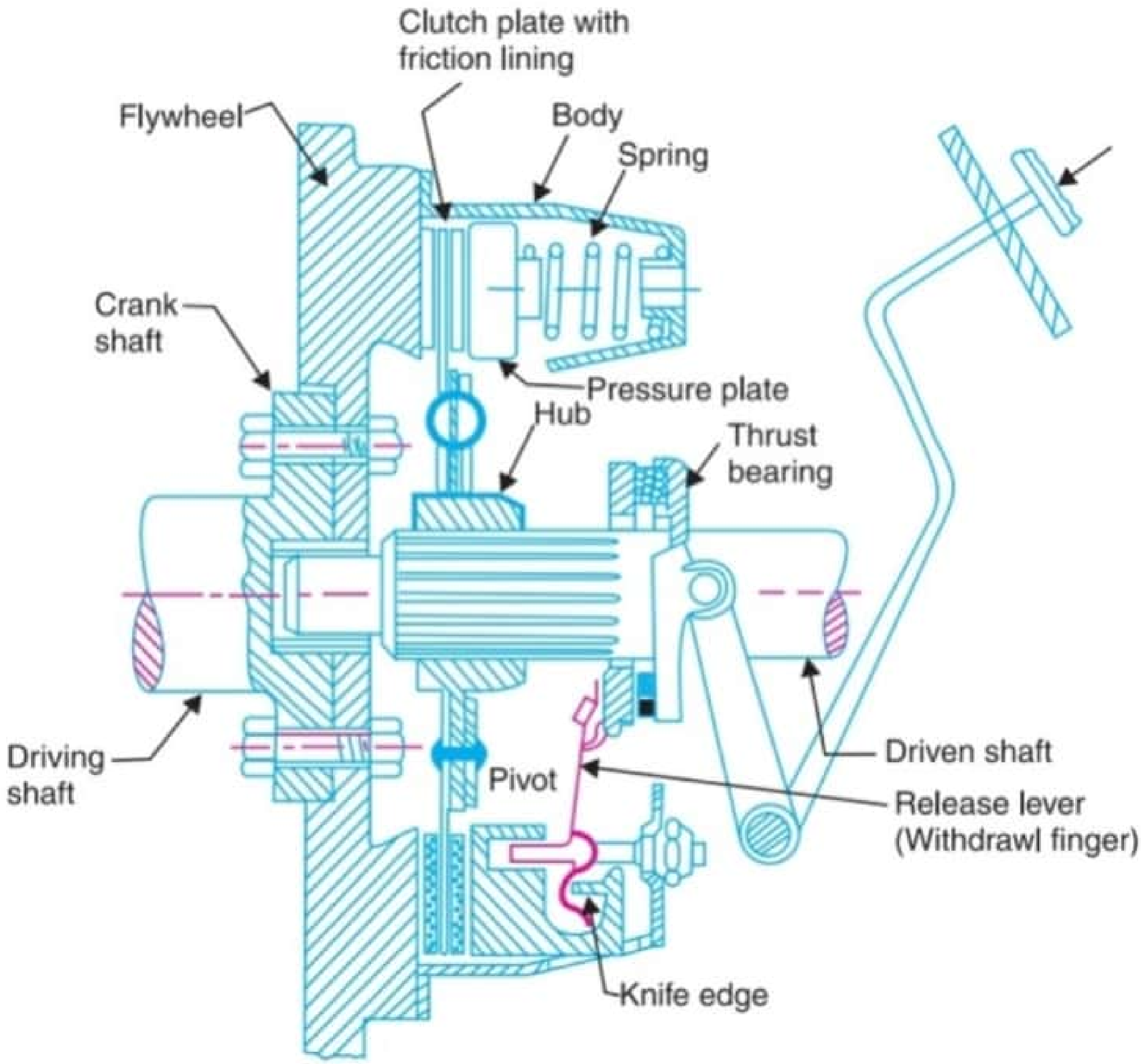
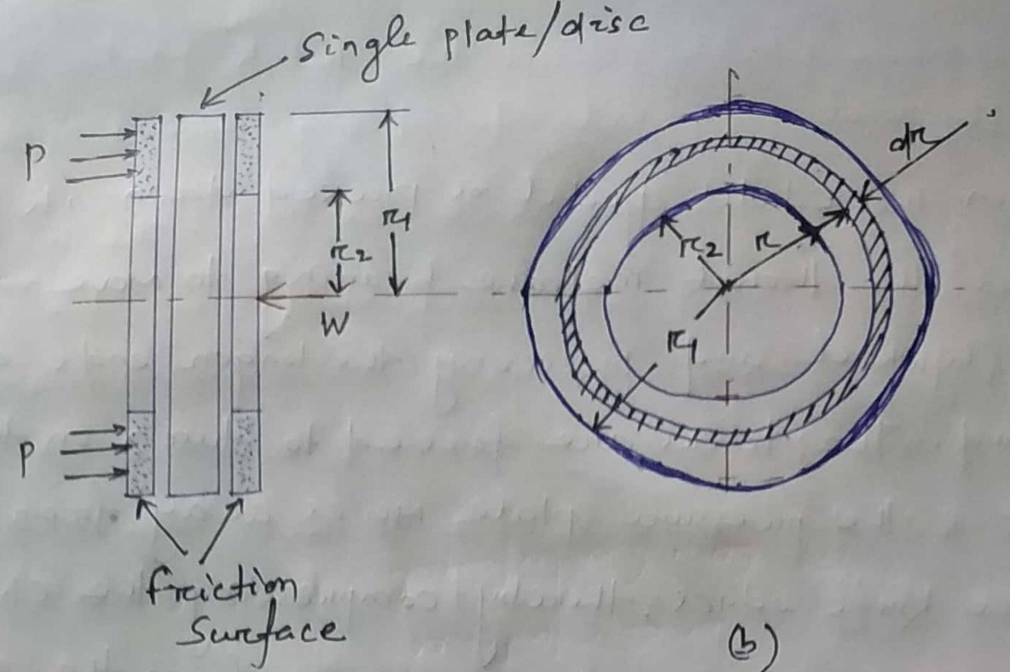


Fig. 10.21. Single disc or plate clutch.

when the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot & the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel & the driven shaft becomes stationary. when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. this allows the springs to extend & thus the pressure plate pushes the clutch plate back towards the flywheel.

The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion bet^n the driving & driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place on the power is transmitted from the driving shaft to the driven shaft.



(a) (b)

forces on a single plate clutch

① Considering uniform pressure:

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

Where W = axial thrust with which the contact or friction surface are held together.

∴ We know that the frictional torque on the elementary ring of radius r & a thickness 'dr' is

$$T_r = 2\pi\mu p . r^2 . dr$$

∴ Total frictional torque acting on the friction surface or on clutch i.e.

$$T = \int_{r_1}^{r_2} 2\pi\mu p . r^2 . dr = 2\pi\mu p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p\left[\frac{r_1^3 - r_2^3}{3}\right]$$

Substituting the value of p then

$$T = 2\pi\mu \times \frac{W}{\pi[r_1^2 - r_2^2]} \times \frac{r_1^3 - r_2^3}{3}$$

$$= \frac{2}{3}\mu W \left[\frac{r_1^3 - r_2^3}{(r_1)^2 - (r_2)^2}\right]$$

$$\Rightarrow \boxed{T = \mu W R}$$

∴ Where R = Mean radius of friction surface

$$\Rightarrow R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

(2) Considering Uniform wear.

Let 'p' is the normal intensity of pressure at a distance 'r' from the axis of the clutch. Since the intensity of pressure varies inversely with the distance.

$$p.r. = C$$

$$\Rightarrow \boxed{p = \frac{C}{r}}$$

∴ Normal force on the ring

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr$$

$$\Rightarrow \delta W = 2\pi C.dr$$

1101153

$\therefore$ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C . dr$$

$$= 2\pi C [r]_{r_2}^{r_1}$$

$$= 2\pi C [r_1 - r_2)$$

$$\Rightarrow \boxed{C = \frac{W}{2\pi (r_1 - r_2)}}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi \mu p . r^2 . dr$$

$$= 2\pi \mu \times \frac{C}{r} \times r^2 . dr \qquad \left(\because p = \frac{C}{r}\right)$$

$$= 2\pi \mu . C . r . dr$$

$\therefore$ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi \mu C . r . dr$$

$$= 2\pi \mu . C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi \mu C \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$= \pi \mu . C \left[r_1^2 - r_2^2\right]$$

$$= \pi \mu \times \frac{W}{2\pi (r_1 - r_2)} (r_1^2 - r_2^2)$$

$$= \frac{1}{2} \mu W (r_1 + r_2)$$

$$\Rightarrow \boxed{T = \mu . W . R}$$

Where R = Mean radius of the friction surface $= \frac{r_1 + r_2}{2}$

Note ① In general total frictional torque acting on the friction surface (or on the clutch) is given by

$$\boxed{T = n \mu W R}$$

Where n = number of pairs of friction or contact surface

R = Mean Radius of friction surface.

② For a single plate/disc clutch, normally both sides of the disc are effective, so a single disc clutch has two pairs of surface in contact i.e. $n = 2$

③ So the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, then $P_{max} \times r_2 = C$ or $P_{max} = \frac{C}{r_2}$

④ If the intensity of pressure is minimum at the outer radius (r_1) of the friction, then

$$P_{min} \times r_1 = C \quad \text{or} \quad P_{min} = \frac{C}{r_1}$$

⑤ In case of a new clutch, the intensity of pressure is approximately uniform but in an old in an old clutch the uniform wear theory is more approximate.

⑥ the average pressure (P_{av}) on the friction or contact surface is given by

$$P_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi (r_1^2 - r_2^2)}$$

(7) The uniform pressure theory gives a higher frictional torque than the uniform wear theory. So that in case of friction clutches, uniform wear should be considered, unless otherwise stated.

Multiple Disc/plate clutch :-

A multiple plate clutch may be used when a large torque is to be transmitted.

The inside disc (made of steel) are fastened to the driver shaft to permit axial motion (except for the last disc)

The outside disc are held by bolts & are fastened to the housing which is keyed to the driving shaft.

→ It is extensively used in Motor Cars, Machine tools etc.

Let n_1 = No. of discs on the driving shaft

n_2 = No. of discs on the driven shaft

∴ Number of pairs of contact surface

$$\boxed{n = n_1 + n_2 - 1}$$

Total frictional torque acting on the friction surface or on the clutch

$$\boxed{T = n \mu W R}$$

Where R = Mean radius of the friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \text{ — for uniform pressure}$$

$$= \frac{r_1 + r_2}{2} \text{ ——— for uniform wear.}$$

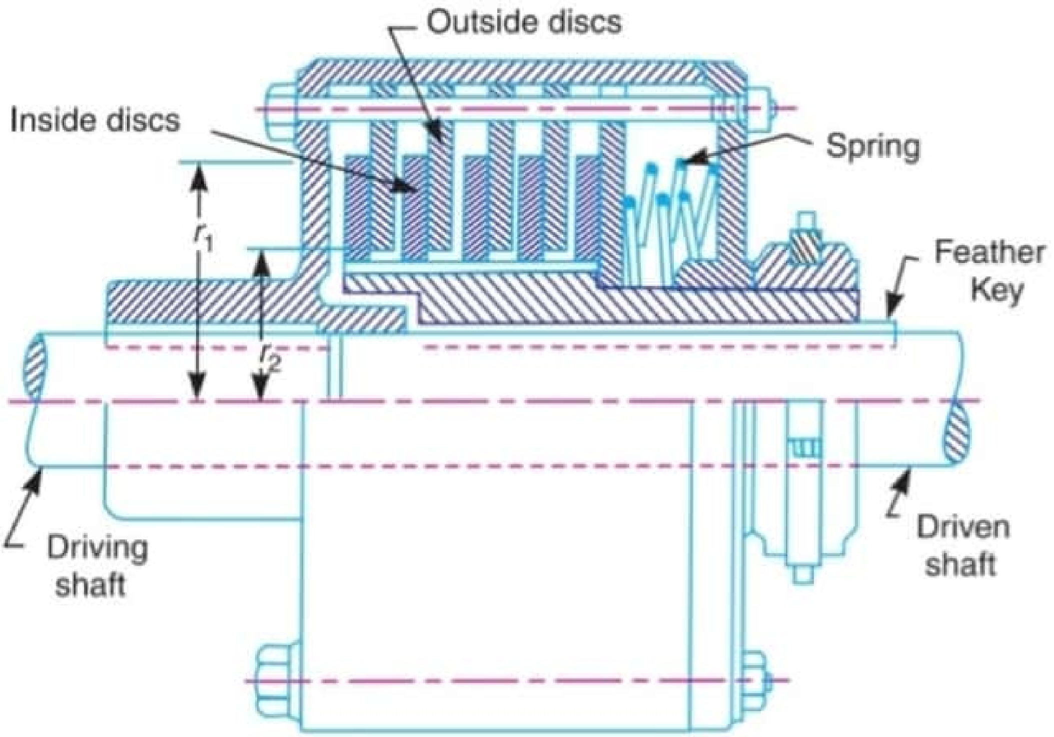


Fig. 10.23. Multiple disc clutch.

Q) Determine the maximum, minimum & average pressure in plate clutch when the axial force is 4 KN. The inside radius of the contact surface is 50mm and the outside radius is 100mm. Assume Uniform wear.

Brake

A brake is a device which is to apply frictional resistance to a moving body to stop or retard it by absorbing its kinetic energy.

→ In all types of motion, there is always some amount of resistance which retards the motion & is sufficient to bring the body rest. So that the time taken and the distance covered in this process is usually too large. By providing Brakes, the external resistance is considerably increased & the period of retardation shortened.

→ The functional difference betⁿ a clutch & a brake is that a clutch is used to keep the driving & driven members moving together, where as brakes are used to stop a moving member or to control its speed.

Types of Brake:

1. Hydraulic Brake
2. Electric Brake
3. Mechanical Brake
   - Radial
     - External Brake
     - Internal Brake
   - Axial
     - Disc Brake
     - Cone Brake

According to the shape of the friction element

Radial Brakes → Block/ shoe Brake.
Radial Brakes → Band Brake

Dynamometer :-

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted & hence the power of engine.

Types of dynamometer.

2 types
① Absorption dynamometers
② Transmission "

Absorption dynamometers:

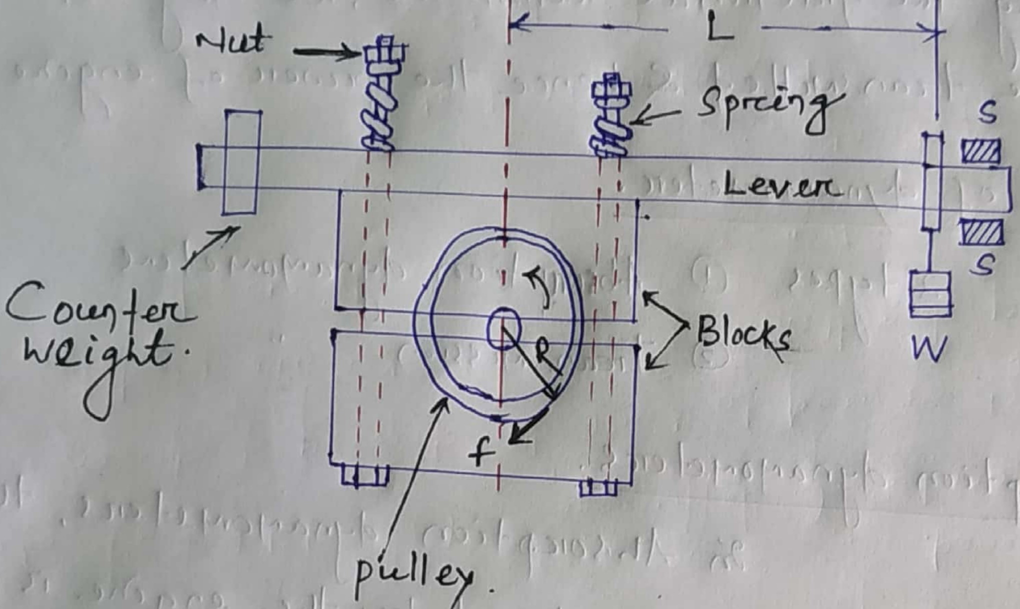
In Absorption dynamometers, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake & is transformed into heat, during the process of measurement.

It is of two types.
① prony brake dynamometer
② Rope brake dynamometer.

① prony brake dynamometer :

This is the simplest type of dynamometer. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to measured. The blocks are clamped by means of two bolts & nuts. A helical spring is provided betn the nut ~~bolt~~ and the upper block to adjust the pressure on the pulley to

Control its speed. The upper block has a long lever attached to it & carries a weight 'W' at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.



When the brakes is to be put in operation, the long end of the lever is loaded with suitable weights 'W' & the nuts are tightened until the engine shaft runs at a constant speed & the lever is in horizontal position. Under these conditions, the moment due to the weight 'W' must balance the moment of the frictional resistance betn the blocks & the pulley.

Let W = Weight at the outer end of the lever in Newton

L = Horizontal ~~weight~~ distance of the weight W from the centre of the pulley in metres.

F = Frictional resistance betn the blocks & the pulley in Newtons.

R = Radius of the pulley in metres.

speed of the shaft in r.p.m

We know that ~~a~~ the moment of the frictional resistance or torque on the shaft

$$T = W \cdot L$$

$$\Rightarrow T = F \cdot R \ \text{N-m.}$$

$\therefore$ Work done in one revolution

= Torque × Angle turned in radians

$= T \times 2\pi$ N-m.

$\therefore$ work done per minute

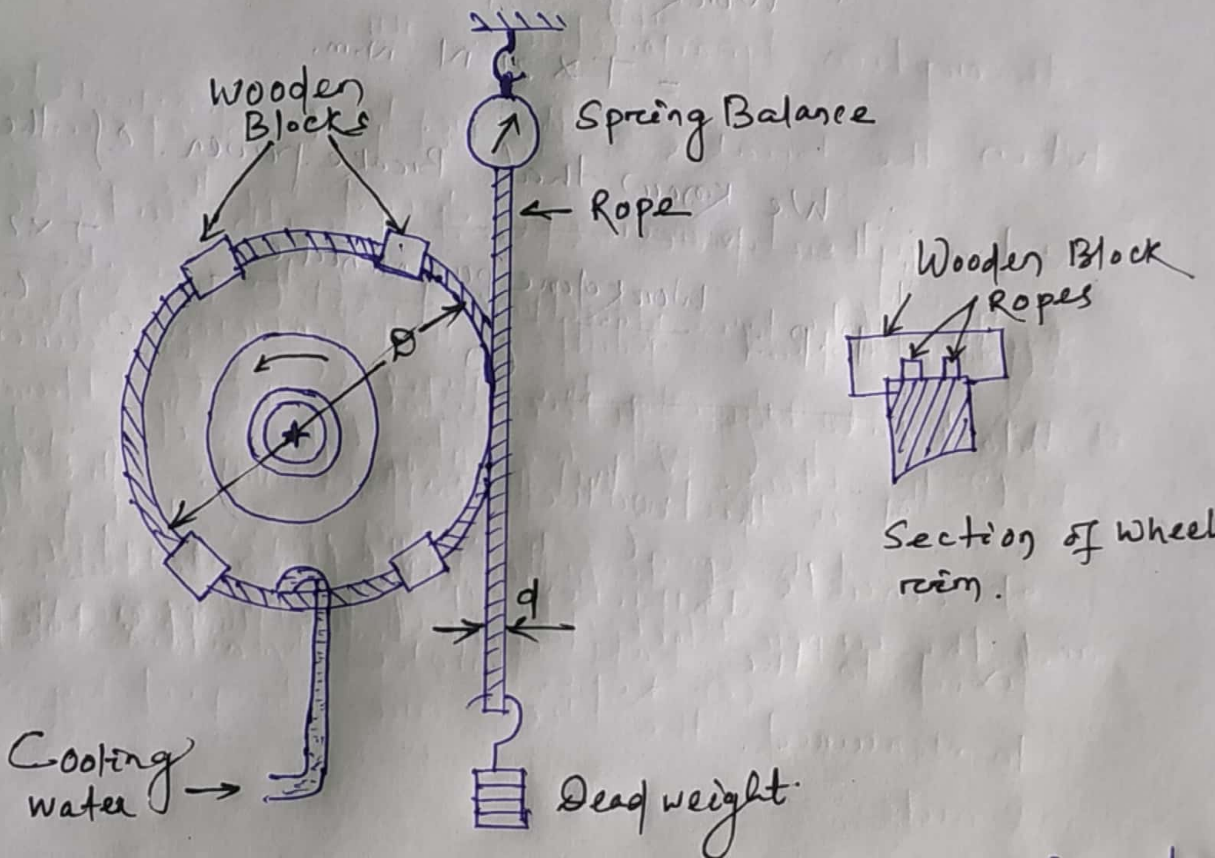
$= T \times 2\pi N$ N-m.

$\therefore$ We know that Brake power of the engine

$$\boxed{B.P = \frac{\text{Work done per minute}}{60} = \frac{T \times 2\pi N}{60} = \frac{W \cdot L \times 2\pi N}{60} \text{ watt.}}$$

It is also used for measuring the Brake power of the engine.

It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper ends of the rope is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.



In the operation of the brake, the engine is made to run at a constant speed. The frictional torque due to the rope, must be equal to the torque being transmitted by the engine.

Let W = Dead load in Newtons.

S = spring balance reading in Newtons.

D = Diameter of the wheel in metres

d = diameter of rope in metres.

N = speed of the engine in r.p.m.

∴ Net load on the brake $= (W-S)$ N.

We know that distance moved in one revolution

$$= \pi(D+d) \text{ m.}$$

∴ Workdone per revolution

$$= (W-S)\pi(D+d) \text{ N-m.}$$

∴ workdone per minute

$$= (W-S)\pi(D+d)N \text{ N-m.}$$

∴ Brake power of the engine,

$$B.P. = \frac{\text{Workdone per minute}}{60}$$

$$\Rightarrow \boxed{B.P. = \frac{(W-S)\pi(D+d)N.}{60} \text{ watt.}}$$

If the diameter of the rope (d) is neglected,

then brake power of the engine

$$\boxed{B.P = \frac{(W-S)\pi D N}{60} \text{ watt}}$$

Q) In a laboratory experiment, the following data were recorded with rope brake.
Diameter of the flywheel 1.2 m, diameter of the rope 12.5 mm.
Speed of the engine 200 rpm, dead load on the brake 600N.
Spring balance reading 150N. Calculate the B.P. of the engine ?

Power transmission.

~~Bet~~ ~~[illegible]~~

Usually, the power transmitted from one shaft to another by means of belts, rope, chains & gears.

→ Belts, rope & chains are used where the distance betn the shaft is large. for small distances, gears are preferred.

→ Belts, ropes & chains are flexible type of connectors i.e. they are bent easily.

→ The flexibility of belts & ropes is due to the property of their materials where-as chain have a number of small rigid elements having relative motion between the two elements.

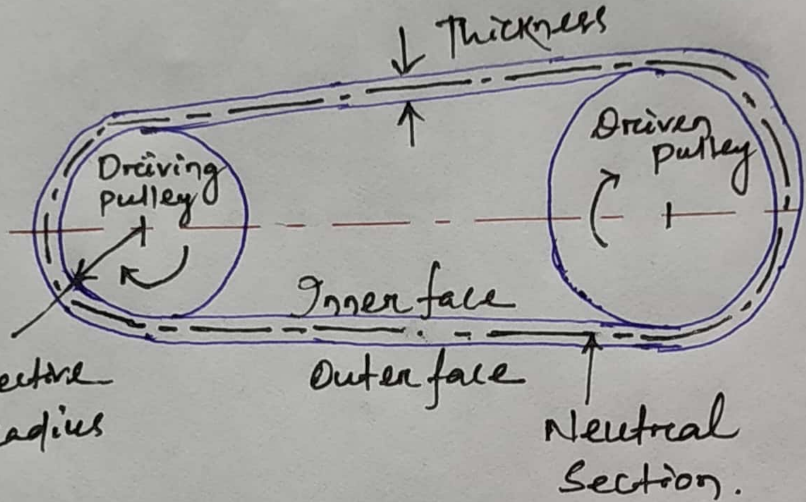
→ Belts & ropes transmit power due to friction betn them & the pulleys. power transmitted exceeds the force of friction the belt or rope slips over the pulley.

→ Belts and ropes are strained during motion as tensions are developed on them.

→ Owing to slipping and straining action, belts & ropes are not positive type of drives, i.e. their velocity ratios are not constant. On the other hand, chains & gears have constant velocity ratios.

Belt drive

To transmit power from one shaft to another, pulleys are mounted on the two shafts. The pulleys are then connected by an endless belt passing over the pulleys the connecting belt is kept in tension so that the motion of one pulley is transferred to the other without slip. The speed of the driven shaft can be varied by varying the diameters of the two pulleys.



For an un-stretched belt mounted on the pulleys, the outer & inner faces become in tension & compression respectively. In between there is a neutral section which has no tension or compression. Usually this is considered at half the thickness of the belt. The effective radius of rotation of a pulley is obtained by adding half the belt thickness to the radius of the pulley.

1101381

Belt drive.

Types of Belt drive.

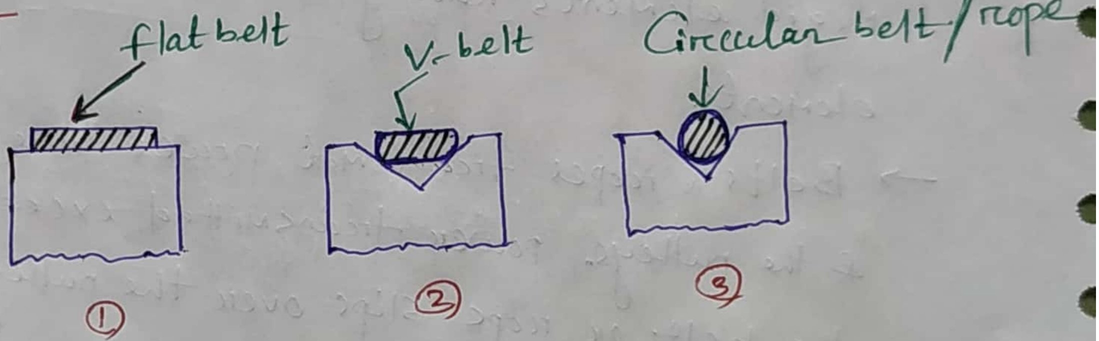
It is 3 types.

① **Light drives** : These are used to transmit small powers at belt speeds upto about 10 m/s.
eg: agricultural machine & small machine tools.

② **Medium drives** : these are used to transmit medium powers at belt speeds upto about 10 m/s to 22 m/s.
eg: Machine tools.

③ **Heavy drives** : These are used to transmit large powers at belt speed above 22 m/s.
eg: compressors & generators.

Types of Belts.



④ **Flat belt** :- It is used where moderate amount of power is transmitted, from one pulley to another pulley when the two pulleys are not more than 8 meters apart.

eg. factories, workshops.

Types of flat belt drive – ① open belt drive ② cross/twist belt drive ③ Quarter turn belt drive ④ belt drive with idler pulleys. ⑤ Compound belt drive. ⑥ Stepped/cone pulley drive ⑦ fast & loose pulley drive

② **V-belt** : It is used where a moderate amount of power is to be transmitted from one pulley to another, when two pulleys are very near to each other.

③ **Circular belt or rope** : It is used where large amount of power is to be transmitted from one pulley to another, when the two pulleys are more than 8 meters apart.

Material used for Belts.

→ The material used for belts & ropes must be strong, flexible and durable.

→ It must have a high co-efficient of friction.

→ Material used in belts such as Lather, cotton or febric, Rubber belts etc.

Velocity Ratio of Belt drive

It is defined as the ratio between the velocities of the driver and the follower or driven.

Mathematically,

Let, d_1 = diameter of the driver

d_2 = diameter of the follower/driven

N_1 = Speed of the driver in r.p.m

N_2 = Speed of the follower in r.p.m.

∴ Length of belt that passes over the driver, in one minute

$= \pi d_1 N_1$

1101142

Similarly, length of the belt that passes over the follower in one minute, $= \pi d_2 N_2$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, i.e.

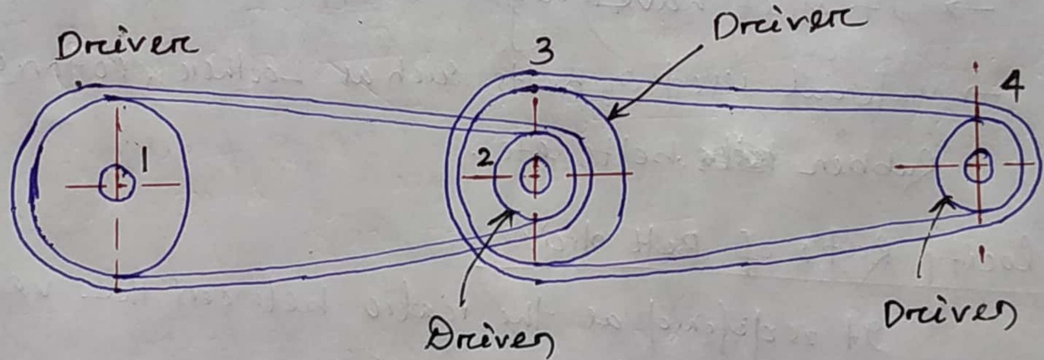
$$\boxed{\pi d_1 N_1 = \pi d_2 N_2}$$

$$\boxed{\therefore \text{ velocity Ratio} \Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}}$$

If the thickness of the belt (t) is considered, then the

$$\boxed{\text{velocity Ratio} \Rightarrow \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}}$$

~~Note~~ Velocity Ratio of a Compound Belt drive:-



Compound belt drive

In compound belt drive the power is transmitted from one shaft to another, through a number of pulleys.

Consider a pulley 1 driving the pulley 2, since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let d_1 = Diameter of the pulley 1

N_1 = Speed of the pulley 1 in r.p.m.

d_2, d_3, d_4 & N_2, N_3, N_4 = Corresponding values for pulleys 2, 3, & 4

We know that velocity ratio of pulleys 1 & 2

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{——— (1)}$$

Similarly, velocity Ratio of pulley 3 & 4

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{——— (2)}$$

Multiplying eqn (1) & (2), we have.

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\Rightarrow \boxed{\frac{N_4}{N_3} = \frac{d_1 \times d_3}{d_2 \times d_4}}$$ ($\because N_2 = N_3$ being keyed to the same shaft)

If there are six pulleys, then

$$\boxed{\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}}$$

i.e. $\dfrac{\text{Speed of the last driven}}{\text{Speed of first driver}} = \dfrac{\text{Product of diameters of drivers}}{\text{product of diameters of drivens}}$

1101141

'Slip' of Belt

We know that the motion of belts & shafts assuming a firm frictional grip between the belts & shafts. But sometimes the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt.

→ It is generally expressed in percentage.

→ The result of the belt slipping is to reduce the velocity ratio of the system.

Let ~~S~~ S_1 = % of slip between the driver & the belt

S_2 = % of slip betn. the belt & the follower/driven

∴ Velocity of the belt passing over the driver per second

$$v = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right) \quad (1)$$

Similarly the velocity of belt passing over the follower per second

$$v_2 = \frac{\pi d_2 N_2}{60} = v - v \times \frac{S_2}{100} = v\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow \frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1}{100} - \frac{S_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1-\frac{S}{100}\right)$$

(Where $S = S_1 + S_2$ i.e. total percentage of slip)

If thickness of the belt (t) is considered, then

$$\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1-\frac{S}{100}\right)}$$

Creep of Belt :-

When the belt passes from slak side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. the relative motion is called **creep**.

→ the total effect of creep is to reduce slightly the speed of the driven pulley or follower.

Considering creep, the velocity ratio is given by

$$\boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_1}}}$$

Where E = Young's modulus for the material of the belt

σ_1 & σ_2 = Stress in the belt on the tight & slack side.

H.W

Q. A shaft runs at 80 rpm & drives another shaft at 150 rpm through belt drive. The diameter of the driving ~~pulley~~ pulley is 600 mm. Determine the diameter of the driven pulley in the following cases.

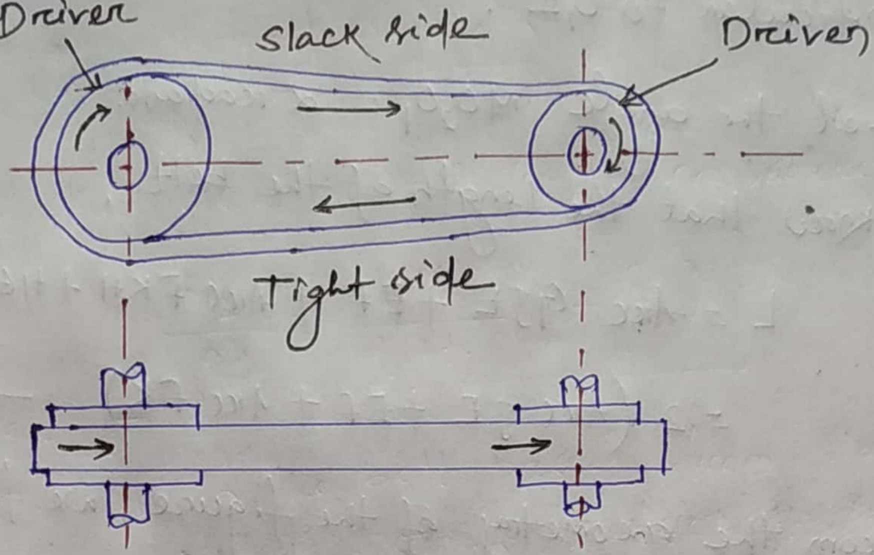
1. Neglecting belt thickness.
2. Taking belt thickness as 5 mm
3. Assuming for case ② a total slip of 4 %.
4. Assuming for case ③ a slip of 2 % on each pulley.

(Ans ⇒ 320 mm, 317.7 mm, 304.8 mm, 304.9 mm)

Q. The power is transmitted from a pulley 1m. diameter running at 200 rpm. to a pulley 2.25 m. diameter by means of a belt. Find the speed lost by the driven pulley as a result of creep, if the stress on the tight & slack side of the belt is 1.4 MPa. & 0.5 MPa respectively. The youngs modulus ~~of~~ for the material of the belt is 100 MPa.

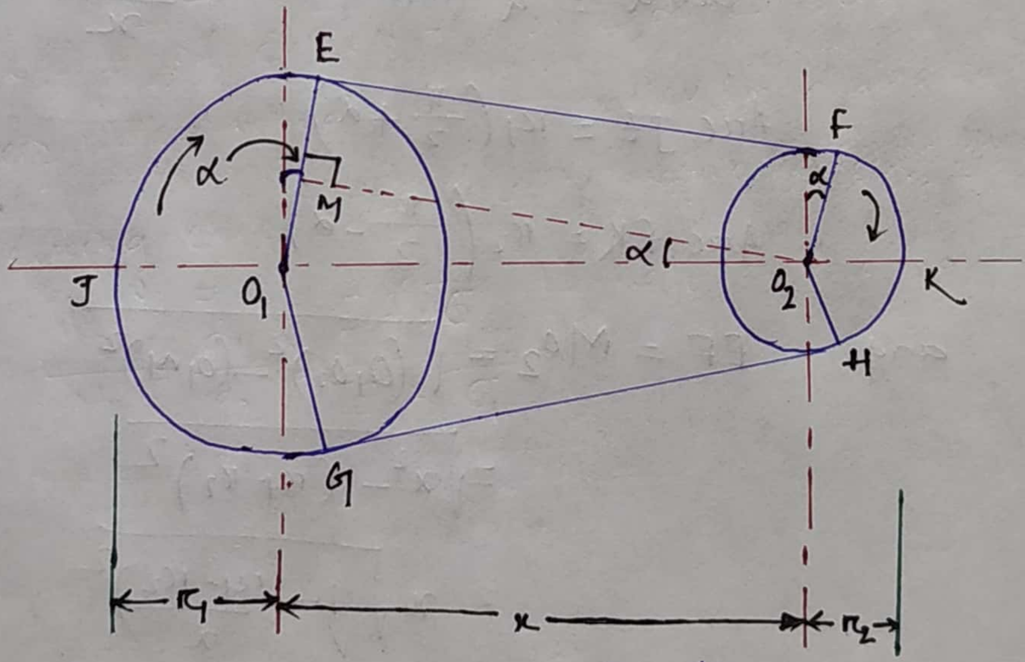
(Ans - 0.2 rpm)

~~Length of~~ open Belt drive.



→ The open belt drive is used with shaft arranged parallel & rotating in the same direction.

Length of an open Belt drive :-



Both the pulleys rotate in the same direction.

Let r_1 & r_2 = radii of the Larger & Smaller pulleys.

x = distance betⁿ the centres of two pulleys (i.e. $O_1 O_2$).

L = Total length of the belt.

Let the belt leaves the larger pulley at 'E' & 'G' & the Smaller pulley at F & H through O_2; draw O_2M parallel to FE. from the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{——— (1)}$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since 'α' is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radian)} = \frac{r_1 - r_2}{x} \quad \text{——— (2)}$$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{——— (3)}$$

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \quad \text{——— (4)}$$

and $$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding the eqn by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \cdots\right]$$

$$= x - \frac{(r_1 - r_2)^2}{\ldots} \quad \text{——— (5)}$$

Substituting the value of arc JE from eqⁿ (3), arc FK from eqⁿ (4) & EF from eqⁿ (5) in eqⁿ (1) we get.

$$L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + x - \frac{(r_1-r_2)^2}{2x} + r_2\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1\cdot\alpha + x - \frac{(r_1-r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2\cdot\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1-r_2) + x - \frac{(r_1-r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1-r_2}{x}$ then

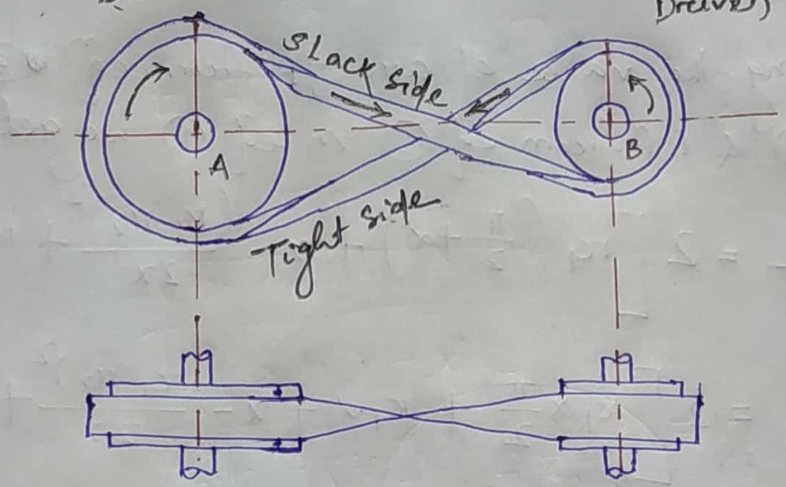
$$L = \pi(r_1+r_2) + 2 \times \frac{(r_1-r_2)}{x} \times (r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\Rightarrow \boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}}$$

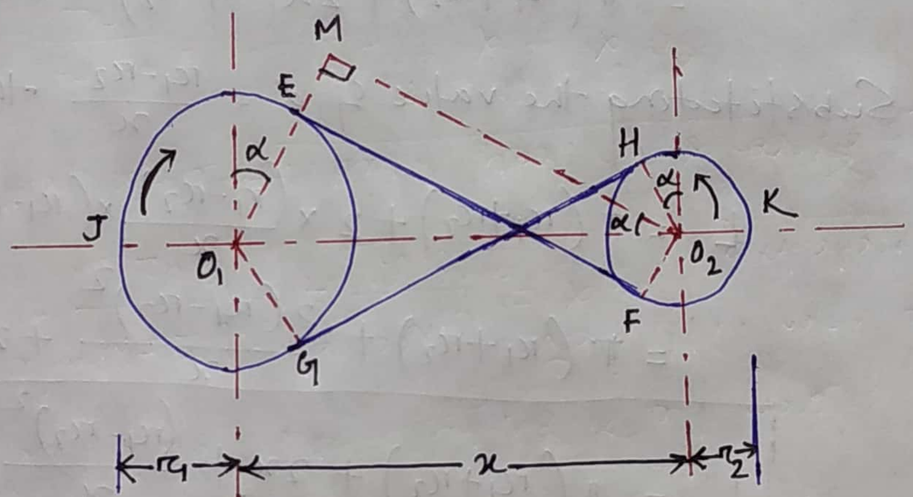
$$\Rightarrow \boxed{L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1-d_2)^2}{4x}}$$

Cross Belt drive



→ It is used with shafts arranged parallel & rotating in the opposite direction.

Length of cross Belt drive :-



Let r_1 & r_2 = Radii of the Larger & Smaller pulleys.

x = Distance bet^n the centres of two pulleys (i.e O_1O_2)

L = Total length of the belt

Let the belt leaves the Larger pulley at E & G & the Smaller pulley at F & H. through O_2, draw O_2M parallel to FE. From the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle MO_2O_1 = α radians.

We know that the length of belt.

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{——— (1)}$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small,

$$\therefore \sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \quad \text{——— (2)}$$

$$\therefore \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{——— (3)}$$

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \quad \text{——— (4)}$$

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this eq^n by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \cdots\right] = x - \frac{(r_1 + r_2)^2}{2x} \quad \text{——— (5)}$$

Substituting the values of eq^n (3), (4) & (5) in eq^n (1) we have

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1 \cdot \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2 \cdot \alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

1101137

Substituting the value of $\alpha = \frac{r_1+r_2}{x}$ then

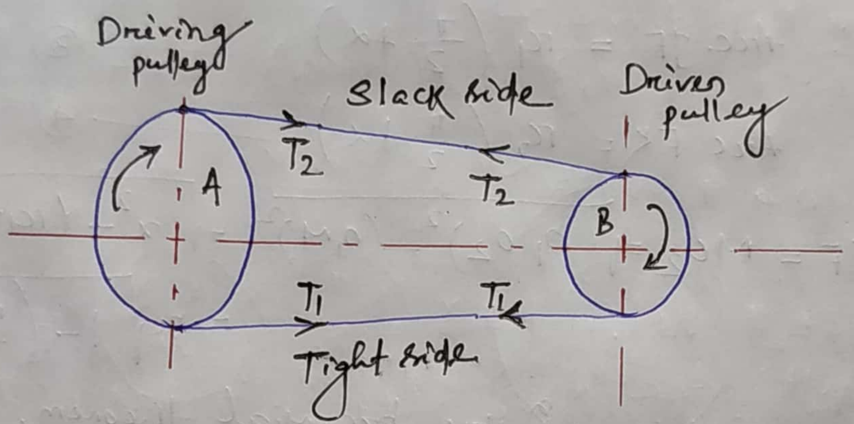
$$L = \pi(r_1+r_2) + \frac{2(r_1+r_2)}{x} \times (r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + \frac{2(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$\Rightarrow \boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}}$$

$$\Rightarrow \boxed{L = \frac{\pi}{2}(d_1+d_2) + 2x + \frac{(d_1+d_2)^2}{4x}}$$

power Transmitted by Belt :—



The driving pulley A & the driven pulley B. we have already discussed that the driving pulley pulls the belt from one side & delivers the same to the other side. It is thus obvious that the tension on the ~~forortera side~~ tight side will be greater than the slack side.

Let T_1 & T_2 = Tensions in the tight & slack side of the belt respectively in Newtons.

r_1 & r_2 = Radii of the driver & follower.

v = velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the follower is the difference betn the two tensions i.e. $(T_1 - T_2)$

$\therefore$ Workdone per Second $= (T_1 - T_2)\,v$ N-m/s.

$\therefore$ power transmitted $P = (T_1 - T_2)\,v$ Watt.

A little consideration will show that the torque on the driving pulley is $(T_1 - T_2)\,r_1$.
Similarly the the torque exerted on the driven pulley i.e. follower is $(T_1 - T_2)\,r_2$

Ratio of Driving tension for flat Belt Drive.

Let the belt leaves the larger pulley at E & G & the Smaller pulley at F & H through O_2; draw O_2M parallel to FE. from the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2(\text{Arc } JE + EF + \text{Arc } FK) \quad \text{——— (1)}$$

from the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since 'α' is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radian)} = \frac{r_1 - r_2}{x} \quad \text{——— (2)}$$

$$\text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \quad \text{——— (3)}$$

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \quad \text{——— (4)}$$

and $$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding the eqn by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \cdots\right]$$

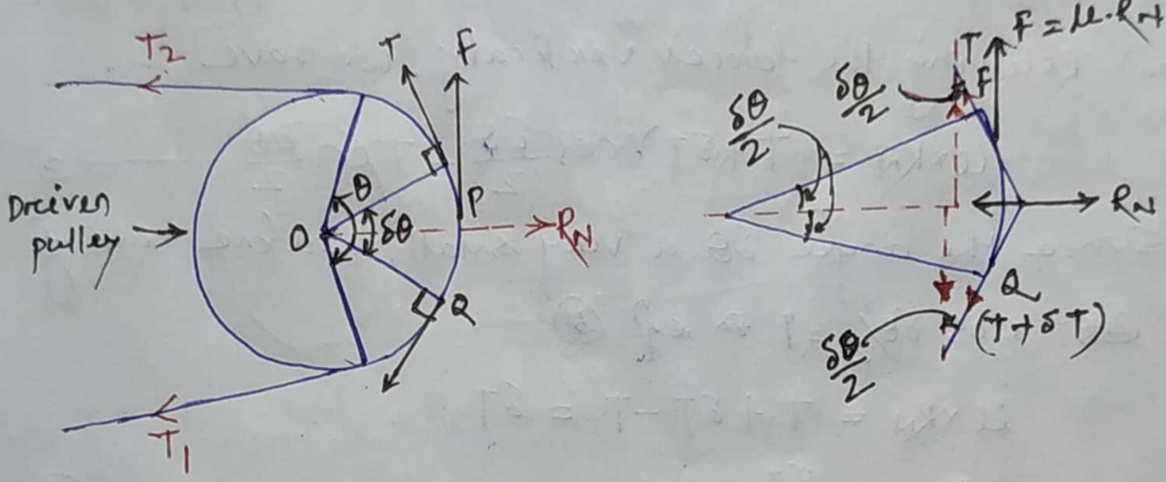
$$= x - \frac{(r_1 - r_2)^2}{2x} \quad \text{——— (5)}$$

$\therefore$ Workdone per Second $= (T_1 - T_2)v$ N-m/s.

$\therefore$ power transmitted $P = (T_1 - T_2)v$ Watt.

A little consideration will show that the torque on the driving pulley is $(T_1 - T_2)r_1$.
Similarly the the torque exerted on the driven pulley i.e. follower is $(T_1 - T_2)r_2$

Ratio of Driving tension for flat Belt Drive.



Let T_1 = Tension in the belt on the tight side
T_2 = Tension in the belt on the slack side.
θ = Angle of contact in radians (i.e. angle subtended by the arc AB, along which the belt touches the pulley at the centre).

Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley. The belt is in equilibrium under the following forces.

① Tension T in the belt at P,
② Tension $(T+\delta T)$ in the belt at Q,

(3) Normal Reaction R_N,

(4) frictional force, $F = \mu \times R_N$,

Where μ = co-efficient of friction betⁿ the belt & pulley.

Resolving all the forces horizontally & equating the same

$$R_N = (T + \delta T) \sin \frac{\delta\theta}{2} + T \sin \frac{\delta\theta}{2} \quad \text{--- (1)}$$

Since the angle $\delta\theta$ is very small, therefore putting

$\sin \frac{\delta\theta}{2} = \frac{\delta\theta}{2}$ in eqⁿ (1)

$$\Rightarrow R_N = (T + \delta T) \frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2}$$

$$= \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2}$$

$$\Rightarrow R_N = T.\delta\theta \quad \text{--- (2)} \qquad \left(\because \text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now Resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T) \cos \frac{\delta\theta}{2} - T \cos \frac{\delta\theta}{2} \quad \text{--- (3)}$$

Since the angle $\delta\theta$ is very small, therefore putting

$\cos \delta\theta/2 = 1$ in eqⁿ (3)

$$\mu \times R_N = T + \delta T - T = \delta T$$

$$\Rightarrow R_N = \frac{\delta T}{\mu} \quad \text{--- (4)}$$

equating the values of R_N from eqⁿ (2) & (4)

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides betⁿ the limits T_2 & T_1 & from 0 to θ respectively i.e.

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\Rightarrow \log_e \left(\frac{T_1}{T_2}\right) = \mu\theta$$

$$\Rightarrow \boxed{\frac{T_1}{T_2} = e^{\mu\theta}} \quad \text{--- (5)}$$

Scanned with CamScanner

∴ Eqn (5) can be expressed in terms of Corresponding Logarithm to the base 10, i.e.

$$\boxed{2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta}$$

The above expression gives the relation betn the tight side & slack side tensions, in terms of coefficient of friction & angle of contact.

Determination of Angle of contact :-

When the two pulleys of different diameters are connected by means of open belt., then the angle of contact or Lap (θ) at the smaller pulley must be taken into consideration

Let r_1 = Radius of Larger pulley
r_2 = Radius of smaller pulley
x = Distance betn centres of two pulleys (i.e. O_1O_2)

We know on open belt drive

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - ME}{O_1O_2} = \frac{r_1 - r_2}{x} \quad (\because ME = O_2F = r_2)$$

$$\boxed{\text{Angle of contact / lap} \quad \theta = (180° - 2\alpha)\frac{\pi}{18} \text{ radian.}}$$

(open belt)

When the two pulleys are connected by means of a crossed belt, then the angle of contact or Lap (θ) on both the pulley is same.

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + ME}{O_1O_2} = \frac{r_1 + r_2}{x}$$

$$\therefore \boxed{\text{Angle of contact or lap} \quad \theta = (180° + 2\alpha)\frac{\pi}{180} \text{ radian}}$$

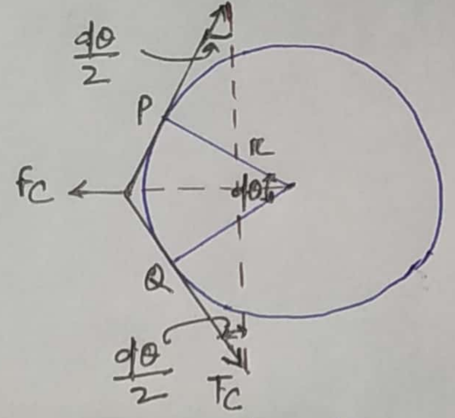
(cross belt)

Q. Find the power transmitted by a belt running over a pulley of 600mm diameter at 200 rpm. The coefficient of friction betn the belt & pulley is 0.30, angle of lap is 160° & Maximum tension in the belt is 2500 N.

Q. Two pulleys, one 450mm diameter & the other 200mm diameter are on parallel shaft 1.95 m. apart & are connected by a crossed belt. find the length of the belt required & the angle of contact betn the belt & each pulley.

What power can be transmitted by the belt when the Larger pulley rotates at 200 rev/min. If the maximum permissible tension in the belt is 1 KN. & the coefficient of friction betn the belt & pulley is 0.25 ?

Centrifugal tension:-



Since the belt continuously runs over the pulleys, therefore some centrifugal force is caused whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension.

→ At Lower belt speed (i.e. less than 10 m/s), the centrifugal tension is very small, but at higher belt speed (more than 10 m/s) its effect is considerable & thus should be taken into account.

Let m = Mass of the belt per unit length in kg
v = Linear velocity of the belt in m/s.
r = Radius of the pulley over which the belt runs in metres.
T_c = Centrifugal tension acting tangentially at P & Q in Newtons.

We know that the length of the belt PQ. $= r.d\theta$

Mass of the belt PQ $= m.r.d\theta$

Centrifugal force acting on the belt PQ

$$f_c = (m r d\theta)\frac{v^2}{r} = m.d.\theta.v^2$$

∴ The centrifugal tension T_c acting tangentially at P & Q keeps the belt in equilibrium.

Now resolving the forces (i.e. centrifugal force & centrifugal tension) horizontally & equating the same we have

$$T_c \sin\left(\frac{d\theta}{2}\right) + T_c \sin\left(\frac{d\theta}{2}\right) = f_c = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression.

$$2T_c\left(\frac{d\theta}{2}\right) = m \cdot d\theta \cdot v^2$$

$$\Rightarrow T_c = m \cdot v^2$$

Note → when the centrifugal tension is taken into account then total tension in the tight side,

$$T_{t1} = T_1 + T_c$$

total tension in the slack side $T_{t2} = T_2 + T_c$

→ power transmitted $P = (T_{t1} - T_{t2})v$ watts.

$$= \left[(T_1 + T_c) - (T_2 + T_c)\right] v$$

$$\Rightarrow P = (T_1 - T_2)\, v.$$

Thus we see that centrifugal tension has no effect on the power transmitted.

→ the ratio of driving tensions may also be written as

$$2.3 \log\left(\frac{T_{t1} - T_c}{T_{t2} - T_c}\right) = \mu \cdot \theta.$$

T_{t1} = maximum / total tension in the belt.

Maximum Tension in the belt

The maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1})

Let σ = Maximum safe stress in N/mm^2

b = width of the belt in mm.

t = Thickness of the belt in mm.

We know that maximum Tension in the belt,

T = maximum stress × Cross-sectional area of belt

$T = \sigma . b . t$

When the centrifugal tension is neglected, then

T or $T_{t1} = T_1$ (i.e. tension in the tight side of the belt)

When the centrifugal tension is considered, then

T or $T_{t1} = T_1 + T_C$

Condition for the Transmission of maximum power :-

We know that power transmitted by a belt.

$$P = (T_1 - T_2) v \quad \text{--- (1)}$$

Where T_1 = Tension in the tight side of the belt in Newtons.

T_2 = Tension in the slack side of the belt in N.

v = velocity of the belt in m/s.

We know that the ratio of driving tension is

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{(or)} \quad T_2 = \frac{T_1}{e^{\mu\theta}} \quad \text{--- (2)}$$

Substituting the value of T_2 in eqn (1)

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) v$$

$$= T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) v \quad \text{--- (3)}$$

$$\Rightarrow P = T_1 . v . C$$

Where $C = 1 - \frac{1}{e^{\mu\theta}}$

We know that $T_1 = T - T_C$

T = Maximum tension to which the belt can be subjected in Newtons.

T_C = Centrifugal tensions.

Substituting the value of T_1 in eqn 3

$$P = (T - T_c) v \cdot C$$
$$= (T - mv^2) v \cdot C$$
$$\Rightarrow P = (Tv - mv^3) C \quad (\because T_c = m \cdot v^2)$$

For maximum power, differentiate the above expression with respect to v & equate to zero.

$$\frac{dP}{dv} = 0$$
$$\Rightarrow \frac{d}{dv}(T \cdot v - mv^3) C = 0$$
$$\Rightarrow T - 3m \cdot v^2 = 0$$
$$\Rightarrow T - 3T_c = 0$$
$$\Rightarrow \boxed{T = 3T_c} \quad \text{---------- (4)}$$

It shows that when the power transmitted is maximum 1/3rd of the maximum tension is absorbed as centrifugal tension.

Note → We know that $T_1 = T - T_c$ & for maximum power,

$$T_c = \frac{T}{3}.$$
$$\therefore \boxed{T_1 = T - \frac{T}{3} = \frac{2T}{3}}$$

→ From eqn (4), the velocity of the belt for the maximum power,

$$\boxed{v = \sqrt{\frac{T}{3m}}}$$

Initial Tension in the belt :-

When a belt is wound round the two pulleys (i.e. driver & follower), its two ends are joined together, so that the belt may continuously move over the pulleys, since the motion of the belt from the driver & the follower is governed by a firm grip, due to friction betn the belt & the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called initial tension.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) & delivers it to the other side (decreasing the tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side & the decreased tension in other side of the belt is called tension in the slack side.

Let T_0 = Initial tension in the belt.

T_1 = Tension in the tight side of the belt

T_2 = Tension in the slack side of the belt

α = Co-efficient of increase of the belt length per unit force

A little consideration will show that the increase of tension in the tight side $= T_1 - T_0$

& increase in the length of the belt on tight side

$$= \alpha(T_1 - T_0) \quad \text{--- (1)}$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

decrease in the length of the belt on the slack side,

$$= \alpha(T_0 - T_2) \quad \text{--- (2)}$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side.

Thus equating eqn (1) & (2)

$$\alpha(T_1 - T_0) = \alpha(T_0 - T_2)$$

$$\Rightarrow T_1 - T_0 = T_0 - T_2$$

$$\Rightarrow \boxed{T_0 = \frac{T_1 + T_2}{2}}$$ (i.e. neglecting Centrifugal tension)

$$\Rightarrow \boxed{T_0 = \frac{T_1 + T_2 + 2T_C}{2}}$$ (considering Centrifugal tension)

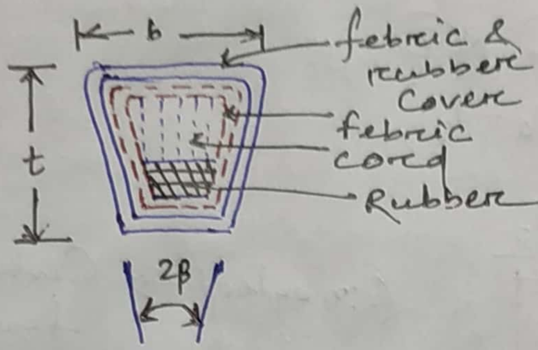
V-belt drive :-

We have already discussed that a V-belt is mostly used in factories & workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.

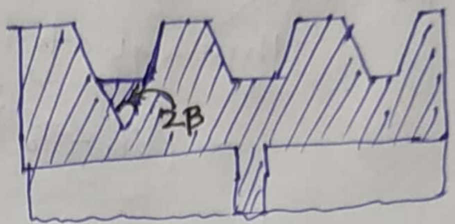
The V-belts are made of febric & cords moulded in rubber & covered with febric & rubber. These belts are moulded to a trapezoidal shape & are made endless. These are particularly suitable for short drives, i.e. when the shafts are at a short distance apart. The included angle of V-belt is usually from 30°–40°.

In case of flat belt drive, the belts runs over the pulleys whereas incase of V-belt drive, the rim of the pulley is grooved in which the V-belt ~~runs~~ runs.

The effect of the groove is to increase the frictional
grip of the v-belt on the pulley & thus reduce the tendency
of slipping. In order to have a good grip on the pulley,
the v-belts is in contact with the side faces of the groove
& not at the bottom. The power is transmitted by the
= wedging action bet^n the belt & the v-groove in the pulley.



a) Cross-Section of a V-belt

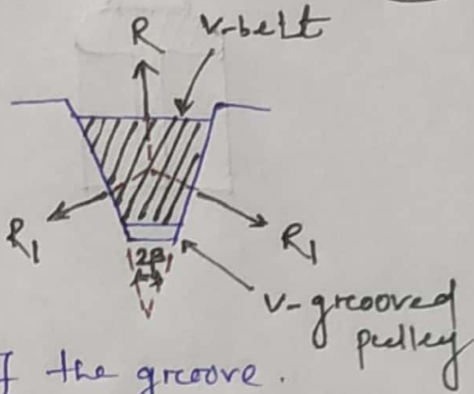


b) Cross-Section of V-grooved pulley

A clearance must be provided at the bottom of
the groove in order to prevent touching to the bottom
as it becomes narrower from wear. The v-belt drive,
may be inclined at any angle with tight side either
at top or bottom. In order to increase the power output
several v-belts may be operated side by side. It may
be noted that in multiple v-belt drive, all the belts
should stretch at the same rate so that the load is
equally divided bet^n them. When one of the set of belt
brakes, the entire set should be replaced at the same
time. If only one belt is replaced, the new unworn and
unstressed belt will be more tightly stretched and
will move with different velocity.

Ratio of driving tensions for V-belt

A V-belt with a grooved pulley as shown.



Let R_1 = Normal reaction betn the belt & side of the groove.

R = Total reaction on the plane of the groove.

2β = Angle of the groove.

μ = Co-efficient of friction betn the belt & sides of the groove.

Resolving the reactions vertically to the groove.

$$R = R_1 \sin\beta + R_1 \sin\beta = 2R_1 \sin\beta$$

$$\therefore R_1 = \frac{R}{2\sin\beta}.$$

We know that frictional force $= 2\mu \cdot R_1$

$$= 2\mu \times \frac{R}{2\sin\beta}$$

$$= \frac{\mu R}{\sin\beta}$$

$$= \mu R \operatorname{cosec}\beta.$$

Consider a small portion of the belt, subtending an angle $\delta\theta$ at the centre. The tension on on side will be 'T' & on the other side '$T + \delta T$'. Now proceeding power transmitted by belt eqn we get the ~~frictional~~ ~~resistance~~ frictional resistance equal to $\mu \cdot R \cdot \operatorname{cosec}\beta$. instead of the frictional resistance $\mu \cdot R$.

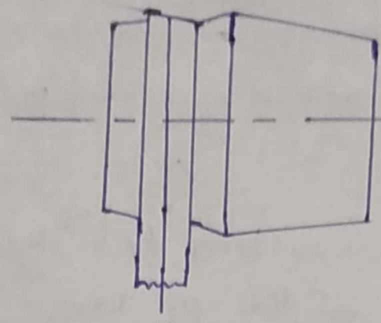
Thus the relation betn the T_1 & T_2 for the V-belt drive will be

$$\boxed{2.3 \log\left(\frac{T_1}{T_2}\right) = \mu \cdot \theta \cdot \operatorname{cosec}\beta.}$$

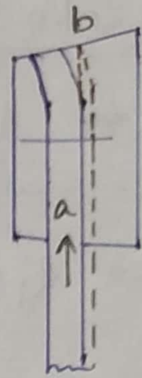
1101130

Crowning of pulleys.

(a)



(b)



(c)

In the fig. (a) the rim of the pulley of a flat belt drive is slightly crowned to prevent the slipping off the belt from the pulley. The crowning can be in the form of conical surface or a convex surface.

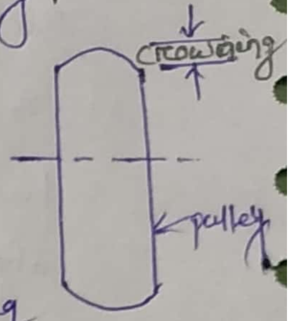
Assume that a belt comes over a conical portion of the pulley & takes the position as shown in fig. (b) i.e. the centre lines remains in a plane, the belt will touch the rim surface at its one edge only. This is impractical. owing to the pull, the belt always ~~stick~~ tends to stick to the rim surface. The belt also has a Lateral stiffness. Thus a belt has to bend in the way shown in the fig (c) to be on the conical surface of the pulley.

Let the belt travel in the direction of the arrow. As the belt touches the cone, the point 'a' on it tends to adhere to the cone surface due to the pull on the belt. This ~~will~~ means as the pulley takes a quater turns, the point 'a' on the belt will be carried to 'b' which is towards the mid plane of the pulley than that previously occupied by the edge of the belt. But again, the belt cannot be stable on the pulley in ~~dotted~~ the upright position & has to bend to stick to the cone surface, i.e. it will occupy the position shown by dotted lines.

Thus, if a pulley is made up of two equal cones or of convex surface, the belt will tend to climb on the slopes & will thus, run with its centre line on the mid-plane of the pulley.

Crowning of pulley

→ In a flat belt ~~drive~~ pulley, the rim surface is given a convex shape by increasing the thickness of a rim at the center. this increased thickness is called crown & the process is known as crowning of pulley.



→ In flat belt drive, if the two shaft are not exactly parallel, there is tendency of belt to come off from the pulley in running condition. The crowning prevents the coming off the belt from the pulley.

→ The crowning helps to keep the belt near the mid plane of the pulley in running condition.

GOVERNORS:-

Introduction:-

speed variation in an engine occurs in two ways:

- cyclic variation
- variation of speed over a number of revolutions

Cyclic variations:-

Cyclic variation occurs because of variation in the turning moment of the engine. These variations can be reduced by mounting a suitable flywheel on the shaft.

Variation of speed over a number of revolutions:-

Variation of speed over a number of revolutions is because of variation of load on the engine.

- In this case a governor is mounted which controls the mean speed of the engine by regulating fuel supply to it.
- When the load increases, speed decreases and it's necessary to increase the fuel supply by opening the throttle valve to maintain mean speed of the engine, and vice versa.

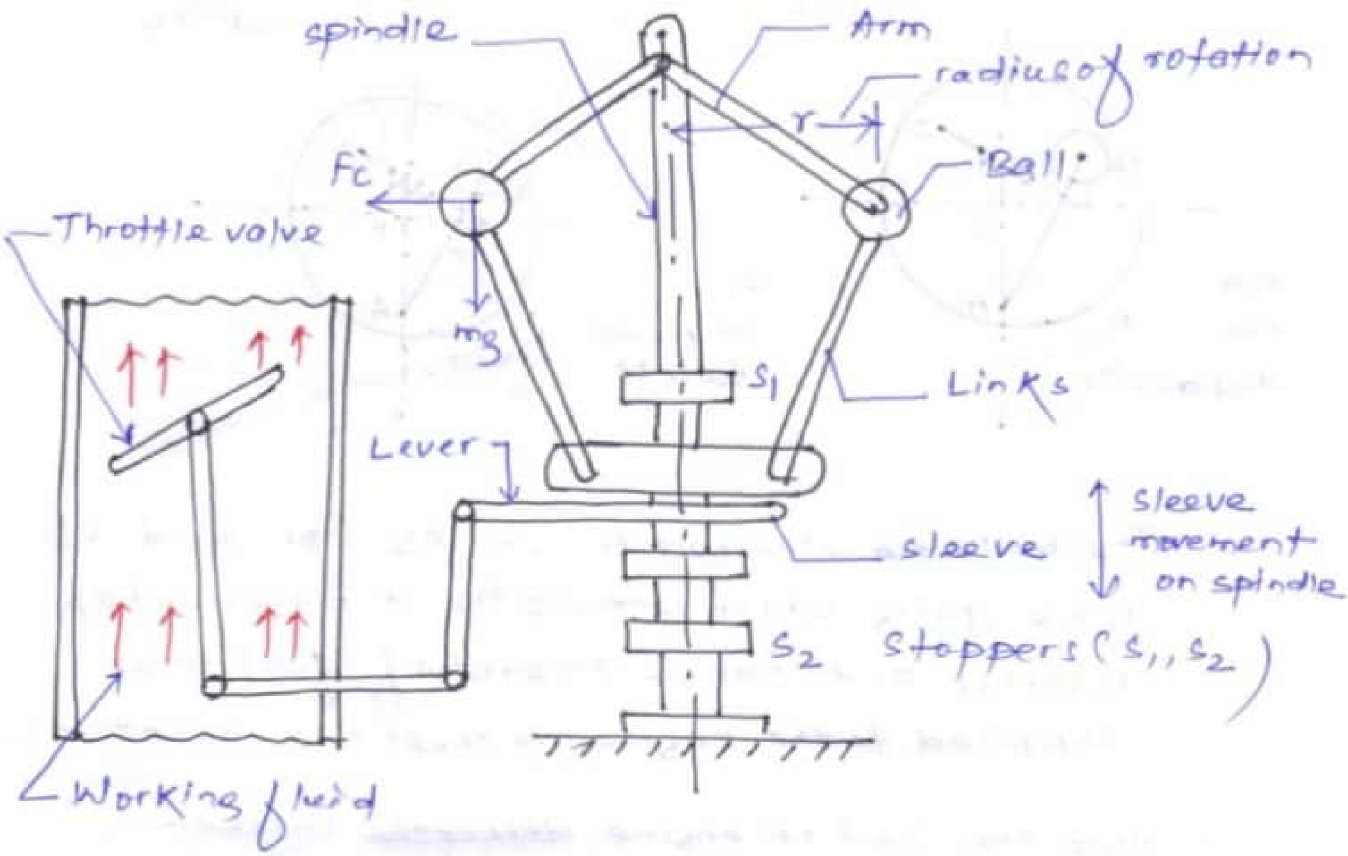
Difference betn Governor and Flywheel:-

Governor	Flywheel
1. Maintains the variation of mean speed within prescribed limit	1. Controls the speed variations in an engine caused due to fluctuations of turning moment.
2. It regulates the speed over a period of time.	2. It regulates the speed during a cycle only.
3. It regulates speed by	

Governor	Flywheel
charge of the engine or prime mover	It stores energy and give it up whenever required in a cycle.

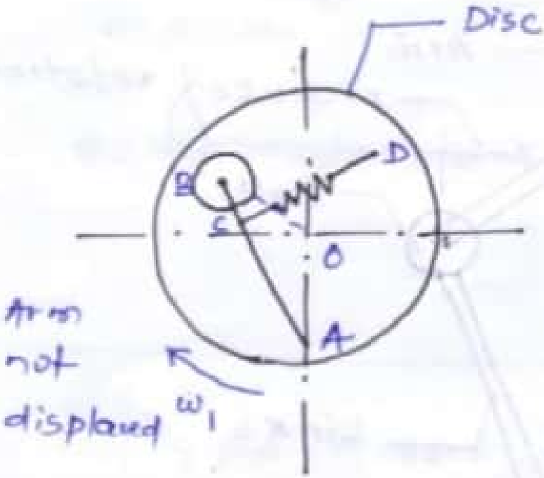
Types of Governor:-

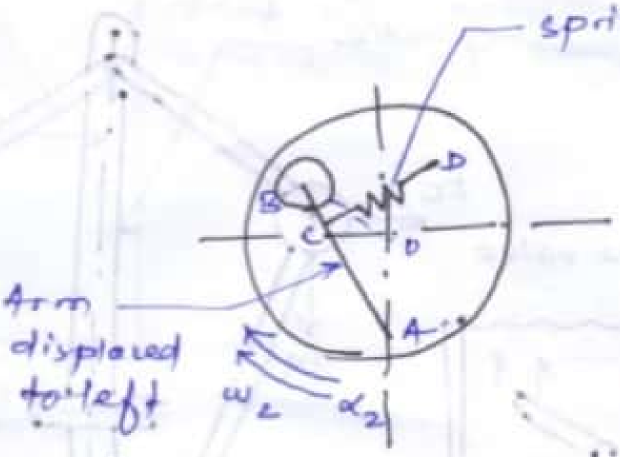
- Governor
  - Centrifugal governor
    - Pendulum type
      - Watt Governor
    - Loaded governor.
      - Dead wt.
      - Spring controlled
        - Hartnell governor
        - Wilson Hartnell governor
        - Hartung
  - Inertia governor.



Centrifugal governor consists of two balls connected to spindle through arms. The upper arms are keyed to the spindle and lower arms (links) are connected to the sleeve. The sleeve is free to slide on the spindle. The balls rotate with spindle (shaft), giving rise to the centrifugal force which radially ~~acts~~ acts outwards. When the speed increases, the balls rotate at a larger radius and the sleeve slides upwards on the spindle, ~~this~~ and with the helps ~~the~~ lever the throttle is closed to the required extent.

- With the decrease in speed the governor ball rotate at smaller radius of rotation, compelling the sleeve to move down on the spindle. The downward movement of sleeve opens the throttle to the required extent to admit the required fuel into





- The ball ~~are~~ is attached to arm AB ~~(and~~ and CD is the spring which controls the displacement of governor and changes amount of fuel to be supplied to the engine to meet the variations.

- When the load on engine ~~decreases~~ increases, the speed of the disc increases to w_2 and is subjected to an angular acceleration α_2 also.

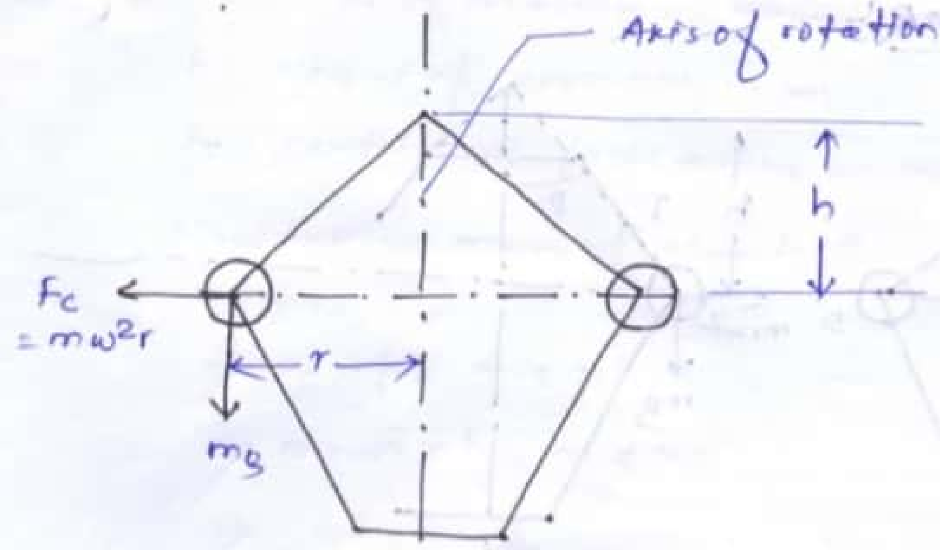
  and $\boxed{w_2 = w_1 + \alpha_2 t}$

  The arm is displaced to left due to centrifugal force on the governor ball and the energy supplied to the engine is cut-off till new equilibrium position is gained.

Terminology :-

following terms are used in governor:-

(a) Height of governor:- vertical distance from centre of ball to point on the spindle axis where the axes of arms intersect.
- it is denoted by 'h'.



(b) centrifugal force:-

centrifugal force $\boxed{F_c = mr\omega^2}$

Where m = mass of the ball in Kg.

r = radius of rotation in m,

ω = angular speed rad/s

(c) controlling force:-

It is an equal and opposite force to that of the centrifugal force.

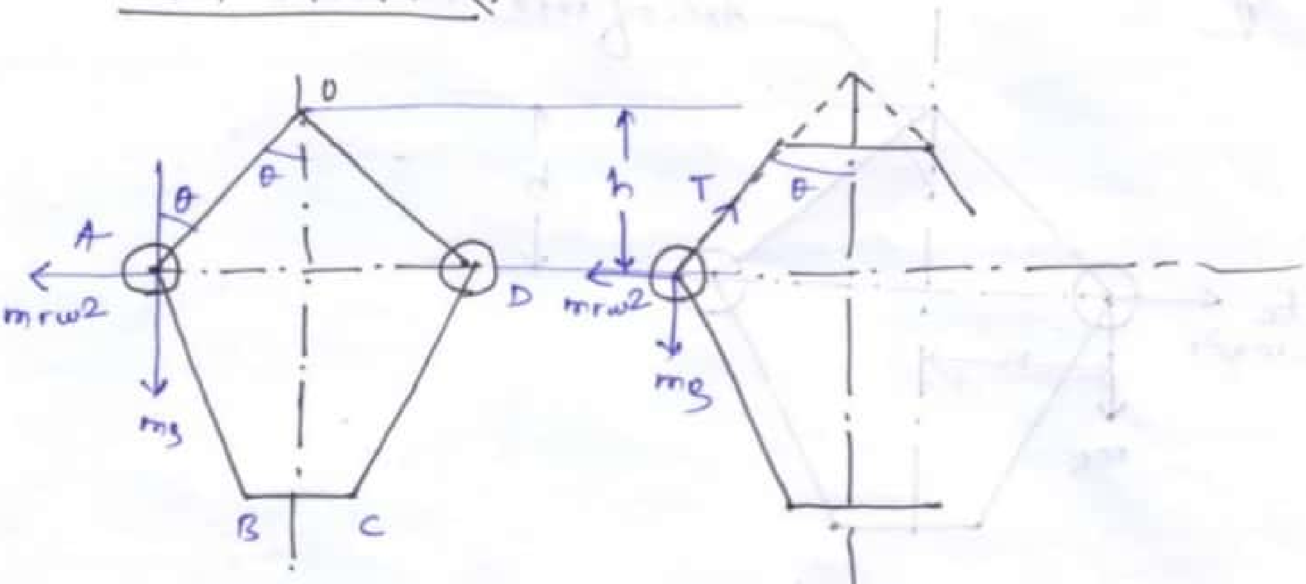
(d) Equilibrium speed:- speed at which governor balls, arms, sleeve etc are in equilibrium and there is no upward or downward movement of sleeve.

(e) Radius of rotation:- It is the horizontal distance between centre of ball and the axis of rotation, denoted by 'r'

(f) Mean equilibrium speed:- It is the speed at ~~of~~ the mean position of the ball or sleeve.

(g) Sleeve lift:- It is the vertical distance travelled by sleeve on the spindle in equilibrium speed.

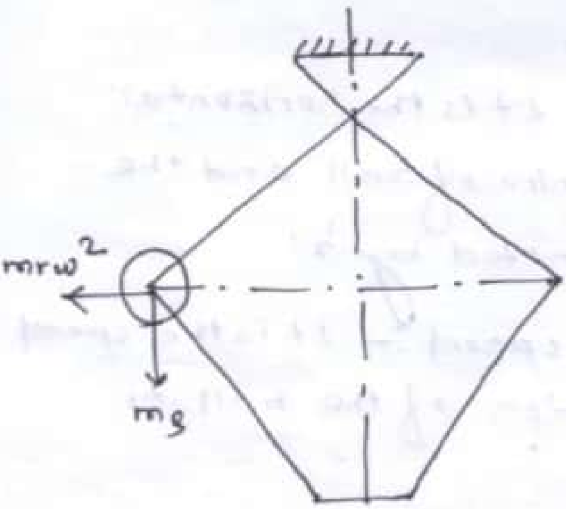
WATT GOVERNOR :-



(a) (b)

Watt governor is the simplest form of centrifugal governor. This governor is named after Watt who used it for steam engines.

- It is basically of three types depending upon the position of upper arms.
- When arms intersect at spindle axis, it is known as pinned arm type watt governor. (fig-a)
- The open-arm and cross-arm type watt governors are shown in fig. (b) and (c)



(c)

- In each of these three cases the lower arms i.e the links are fixed to the sleeve.
- When speed increases the ball moves outwards due to centrifugal force and pull the sleeve upwards on the spindle through the links and the vice versa.

Let m = mass of each ball in Kg.

ω = angular velocity of the balls, about the spindle axis in rad/s.

r = radius of rotation of balls in m

h = height of governor in m.

F_c = centrifugal force acting on the balls, in N.

Now taking moment about O

$\Sigma M_O = 0$

$\Rightarrow F_c \times h = m \cdot g \times r$

$\Rightarrow m r \omega^2 \times h = m \cdot g \times r$

$\Rightarrow \omega^2 = \frac{g}{h}$

$\Rightarrow \left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h}$

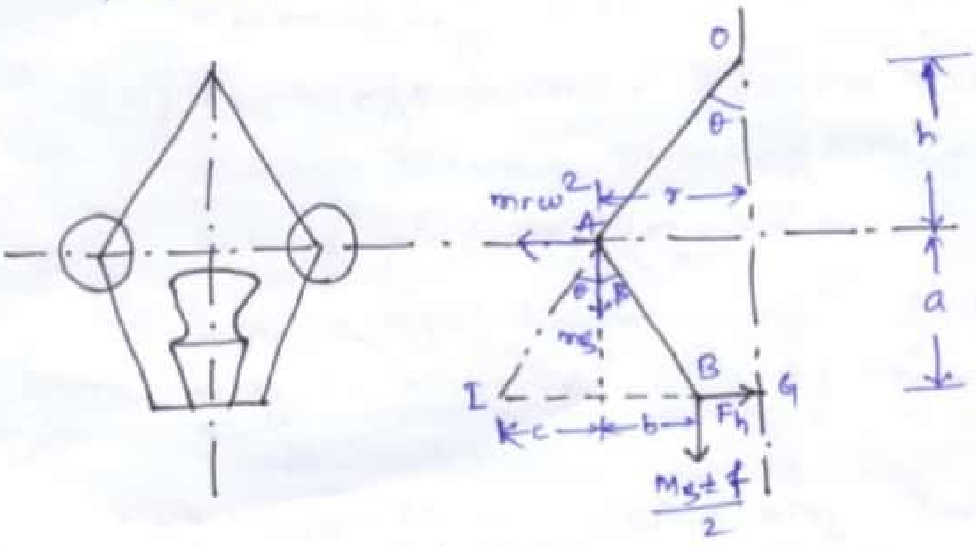
$\Rightarrow N^2 = \frac{9.81 \times 60^2}{(2\pi)^2 \times h} = \frac{895}{h}$

$\Rightarrow \boxed{h = \frac{895}{N^2}}$ m.

from the above equation it's clear that 'h' is inversely proportional to speed 'N' of governor.

PORTER GOVERNOR :-

The modification of Watt's governor with a central load attached to the sleeve is known as porter Governor.



Let M = mass of sleeve

m = mass of each ball

f = force of friction of ~~each ball~~ at the sleeve

The force of friction always acts in a direction opposite to that of the motion. When the sleeve moves up, the force of friction acts in downward direction and total downward force acting on the sleeve is $(Mg+f)$. Similarly when the sleeve moves down, the force on the sleeve will be $(Mg-f)$. In general the net force acting on sleeve is $(Mg \pm f)$

Let h = ht. of the governor

r = distance of center of each ball from the spindle axis.

- Now from the geometry $\triangle$ BAO is a kinematically equivalent to a slider crank mechanism with B as slider (vertical motion), the instantaneous centre of rotation of the link AB is at I for the given configuration of the governor.

- considering equilibrium of left hand half of the governor and taking moment about I

$$\Sigma M_I = 0$$

$$mr\omega^2 \times a = mg \times c + \frac{Mg \pm f}{2}(c+b)$$

$$\text{or } mr\omega^2 = mg \times \frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{c}{a}+\frac{b}{a}\right)$$

$$= mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

where $K = \frac{\tan\beta}{\tan\theta}$

From the above equation

$$mr\omega^2 = \frac{r}{h}\left[mg + \frac{Mg \pm f}{2}(1+K)\right]$$

$$\Rightarrow \boxed{h = \frac{g}{\omega^2} + \frac{(Mg \pm f)(1+K)}{2m\omega^2}}$$

Q.1 A Watt governor runs at 100 rpm. Determine the height of the governor. If the speed of governor increases to 102 rpm find the change in vertical height.

Given: $N_1 = 100$ rpm $\quad N_2 = 102$ rpm.

initial height $h_1 = \frac{895}{N_1^2} = \frac{895}{(100)^2} = 0.0895\,m$

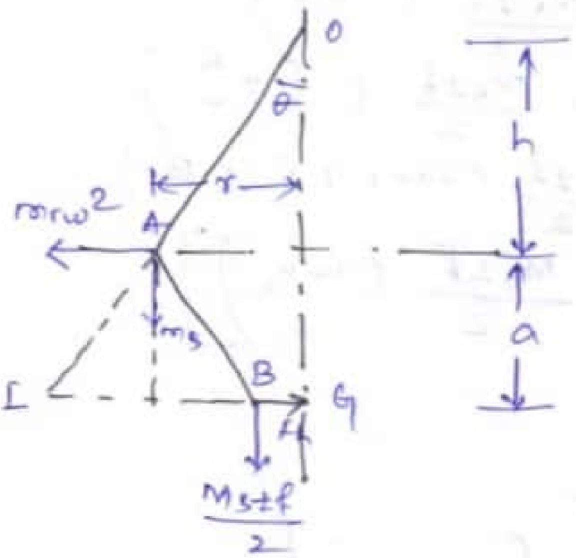
final height $h_2 = \frac{895}{(102)^2} = 0.086\,m$

Change in vertical height $= h_1 - h_2$

$= 0.0895 - 0.0860$

$= 0.0035\,m = \boxed{3.5\,mm}$

Q.2 In a porter governor, each arm is 400 mm long. The lower arms are attached to the sleeve at a distance 45 mm from the axis. Each ball has a mass of 8 kg and the load on the sleeve is 60 kg. What will be the equilibrium speeds for two extreme radii of 250 mm and 300 mm of rotation of governor.



Given data:-

$m = 8\,kg \quad M = 60\,kg.$

$BG = 45\,mm \quad OA = 400\,mm$

Now we have

$mr\omega^2 = \tan\theta\left[mg + \frac{Mg}{2}(1+k)\right]$

(∵ force of friction neglected)

(i) when $r = 250\,mm$

$\tan\theta = \frac{r}{h} = \frac{250}{\sqrt{400^2 - 250^2}} = 0.8$

$K = \frac{\tan\beta}{\ldots} \quad b/a \quad (250-45)/a$

Now $a = \sqrt{(AB)^2 - b^2}$

$= \sqrt{(400)^2 - (205)^2} = 343.4\ mm$

So $K = \dfrac{b/a}{0.8} = \dfrac{205/343.4}{0.8} = \boxed{0.746}$

So we have

$8 \times 0.25 \times w^2 = 0.8 \left[8 \times 9.81 + \dfrac{60 \times 9.81}{2} (1 + 0.746) \right]$

$\Rightarrow 2w^2 = 0.8 (78.48 + 513.85)$

$\Rightarrow w^2 = 237$

or $w = 15.39$

Now $\omega = \dfrac{2\pi N}{60} = 15.39$

$\Rightarrow N = \dfrac{15.39 \times 60}{2\pi} = \boxed{147\ rpm}$

(ii) when radius ~~is~~ = 300 mm

$\tan\theta = \dfrac{r}{h} = \dfrac{300}{\sqrt{400^2 - 300^2}} = 1.134$

$b = 300 - 45 = 255\ mm$

$a = \sqrt{400^2 - 255^2} = 308.2\ mm$

$K = \dfrac{\tan\beta}{\tan\theta} = \dfrac{b/a}{1.134} = \dfrac{(255/308.2)}{1.134} = 0.73$

So,

$8 \times 0.3 \times w^2 = 1.134 \left[8 \times 9.81 + \dfrac{60 \times 9.81}{2} (1 + 0.73) \right]$

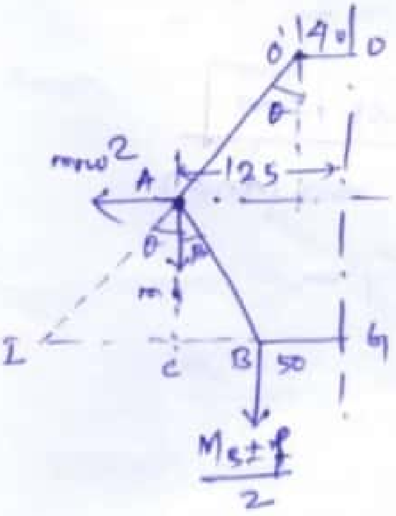
$\Rightarrow 2.4w^2 = 1.134 (78.48 + 509.139)$

$\Rightarrow w^2 = 277.6$

$\Rightarrow w = 16.66 = \dfrac{2\pi N}{60}$

$\Rightarrow \boxed{N = 159.1\ rpm}$

Q.2 Each arm of a porter governor is 250 mm long. The upper arm and lower arm are pivoted to links of 40 mm and 50 mm respectively from axis of rotation. Each ball has a mass of 5 kg and sleeve mass is 50 kg. The force of friction on sleeve mechanism is 40N. Determine the range of speed of the governor for extreme radii of rotation of 125 mm and 150 mm.



Given data:-

arm length = 250 mm

$OO' = 40$ mm, $BG = 50$ mm

$m = 5$ kg, $M = 50$ kg

$f = 40$N, $r_1 = 125$ mm

$r_2 = 150$ mm.

(a) when $r_1 = 125$ mm

$$\tan\theta = \frac{(125-40)}{\sqrt{250^2 - 85^2}} = \frac{85}{235.1} = .361$$

$$\Rightarrow \boxed{\theta = 19.87°}$$

$$\tan\beta = \frac{125-50}{\sqrt{250^2 - 75^2}} = \frac{75}{238.48} \Rightarrow \boxed{\beta = 17.47°}$$

$$\Rightarrow \tan\beta = .215$$

$$K = \tan\beta / \tan\theta = \boxed{.872}$$

Using the relation

$$mr\omega^2 = \tan\theta\left[mg + \frac{Mg - f}{2}(1+K)\right]$$ (∵ as the sleeve moves down, force of friction act upward)

$$\Rightarrow 5 \times 0.125 \times \omega^2 = 0.361\left[5 \times 9.81 + \frac{50 \times 9.81 - 40}{2}(1 + .872)\right]$$

$$\Rightarrow 0.625\omega^2 = 169.929$$

$$\Rightarrow \omega = 16.489$$

$$\Rightarrow N = \frac{16.489 \times 60}{2\pi} = \boxed{157.45 \text{ rpm}}$$

(b) when $r = 150$ mm

$$\tan\theta = \frac{150-40}{\ldots} = \frac{110}{\ldots} = 0.49$$

Scanned with CamScanner

$$\tan\beta = \frac{150-50}{\sqrt{250^2-100^2}} = \frac{100}{229.128} = 0.44$$

$$K = \frac{\tan\beta}{\tan\theta} = \frac{0.44}{0.49} = \boxed{0.897}$$

Now, we have

$$mr\omega_2^2 = \tan\theta\left[mg + \frac{Mg+f}{2}(1+k)\right]$$

$$\Rightarrow 5\times0.15\times\omega_2^2 = 0.49\left[5\times9.81 + \frac{50\times9.81+90}{2}(1+0.897)\right]$$

$$\Rightarrow 0.75\omega_2^2 = 270.592$$

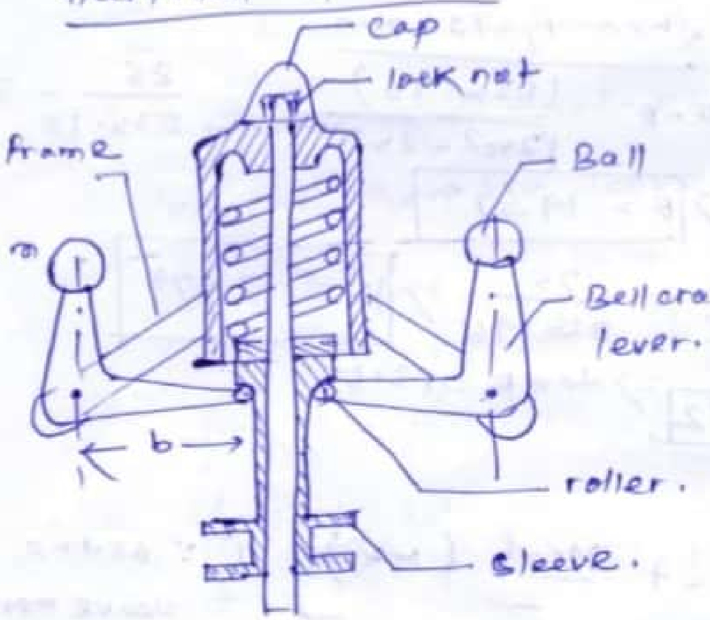
$$\Rightarrow \omega_2 = 18.99 \simeq 19$$

$$\Rightarrow N_2 = 181.38 \text{ rpm.}$$

Range of speed $= N_2 - N_1 = 23.93 \simeq \boxed{24 \text{ rpm}}$

(Ans)

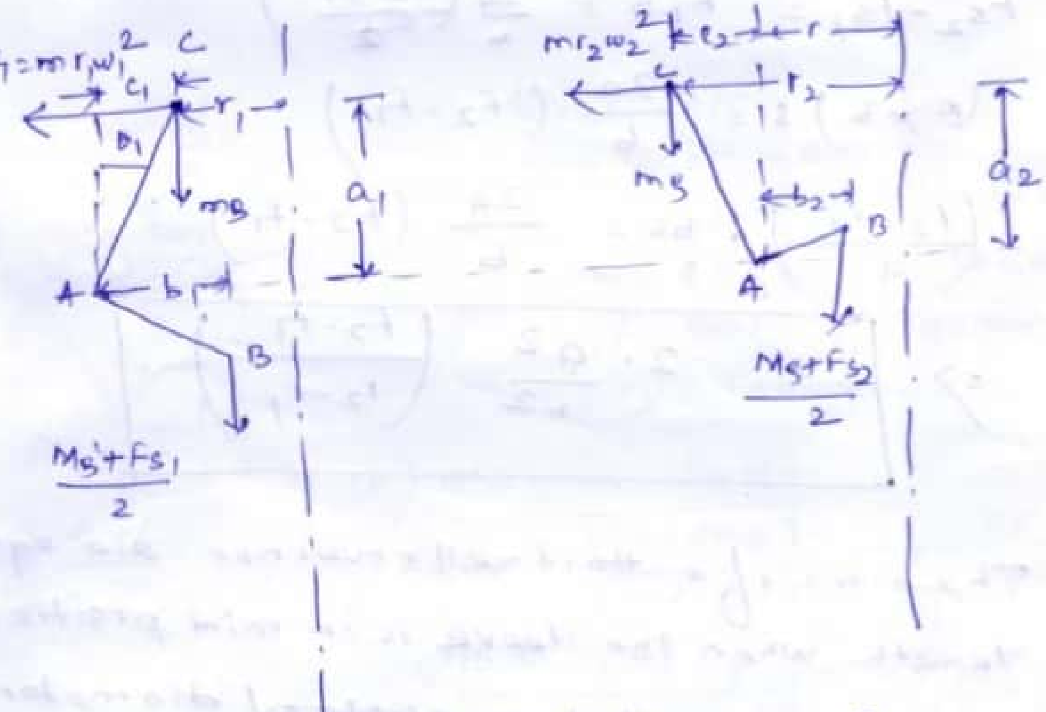
Hartnell Governor:-



Hartnell governor is as shown in the figure. The frame is keyed to the spindle and rotates with it. A compressed spring is placed on the sleeve so that it can exert on sleeve. Two bell crank levers, each carrying a ball at one end and a roller at the other end, are pivoted to a pair of arms. The rollers are fitted into the groove in the sleeve. When speed increases balls move outward compelling the sleeve to slide on

the spindle upward against the spring force.
If the force decreases, the sleeve moves downward
The spring force is adjusted with the help of
locknut. The movement of sleeve is communicated
to the throttle to perform necessary task.

The ~~three~~ positions of bell crank levers are shown
in the figure.



Let F = centrifugal force $= mr\omega^2$

F_S = spring force,

Now taking moment about fulcrum A, $\Sigma M_A = 0$.

$$f_1 a_1 = mg\, c_1 + \left(\frac{Mg+Fs_1}{2}\right) b_1$$

$$F_2 a_2 = mg\, c_2 + \left(\frac{Mg+Fs_2}{2}\right) b_2$$

[Neglecting friction force f] —— (i)

In the working range of governors, θ is very small and so the obliquity effect may be neglected And we have

$a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = c_2 = 0$.

So, $F_1 a = \frac{Mg+Fs_1}{2} \cdot b$ —— (ii) substracting equation (ii) from (iii)

$$(F_2-F_1)\,a = \left(\frac{F_{s_2}-F_{s_1}}{2}\right) b.$$

or $$\boxed{F_{s_2}-F_{s_1} = \frac{2a}{b}\cdot(F_2-F_1)}$$

Now let s = stiffness of spring

h_1 = movement of sleeve.

$$F_{s_2}-F_{s_1} = h_1 s = \frac{2a}{b}\left(\frac{F_2-F_1}{\cancel{2}}\right)$$

$$(\theta \times b)\, s = \frac{2a}{b}(F_2-F_1)$$

$$\left(\frac{r_2-r_1}{a}\right)\cdot bs = \frac{2a}{b}(F_2-F_1)$$

$$\Rightarrow \boxed{s = 2\cdot\frac{a^2}{b^2}\left(\frac{F_2-F_1}{r_2-r_1}\right)}$$

Q. The arms of a Hartnell governor are equal length. When the sleeve is in mid position, the masses rotate a circle of diameter 150 mm Neglecting friction the equilibrium speed for this position is 360 rpm. Max^m variation of speed taking into account friction, is ±6% of mid-position speed for a max^m sleeve moment 30 mm. sleeve mass is 5 kg and friction at the sleeve is 35 N.

– Assuming the power of governor is sufficient to overcome the friction by 1% of change of speed on each side of mid-position find

(i) mass of rotating mass of ball, (ii) spring

Q.3 For a Hartnell governor, following data are provided:

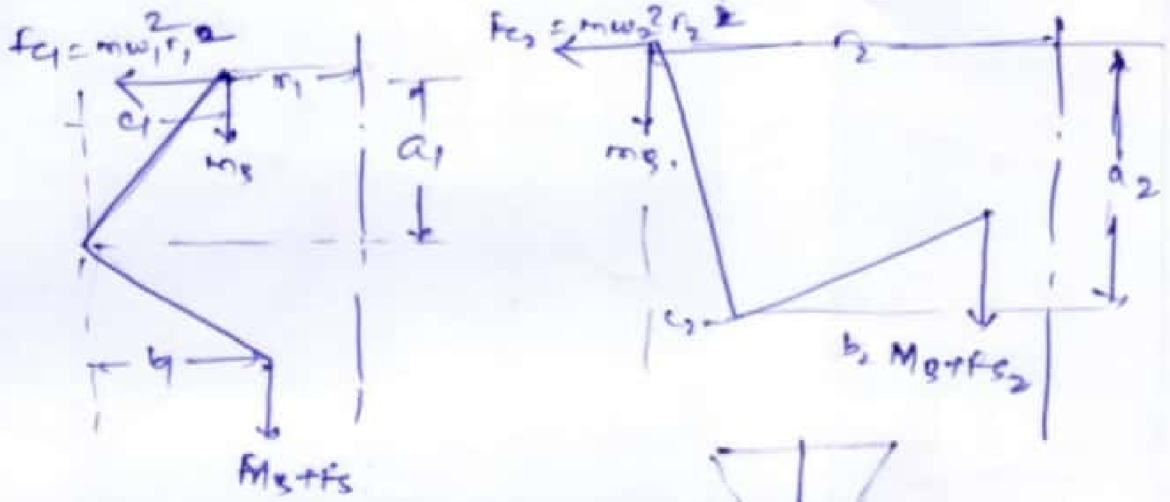
mass of each ball = 1.8 Kg., M = 50 Kg.

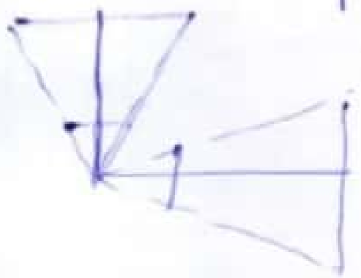
length of vertical arm of ball crank lever = 8.75 cm.

length of other arm of lever = 10 cm.

Speed corresponding to radii of rotations of 12 cm and 13 cm are 296 rpm and 304 rpm respectively. Determine spring stiffness.

Ans :- $m = 1.8$ Kg. $M = 50$ Kg.





$r_1 = 12$ cm $\quad r_2 = 13$ cm.

$$\omega_1 = \frac{2\pi \times 296}{60} = 30.99 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi \times 304}{60} = 31.83 \text{ rad/s.}$$

$$f_{c1} = m\omega_1 r_1{}^2 = 1.8 \times 30.99^2 \times (.12) = \cancel{0.803}$$

$$= 207.44 \text{ N}$$

$$f_{c2} = 1.8 \times 31.83^2 \times .13 = 237.07 \text{ N.}$$

$$\text{spring stiffness } s = 2 \cdot \frac{a^2}{b^2}\left(\frac{f_{c2} - f_{c1}}{r_2 - r_1}\right)$$

$$= 2 \times \frac{0.0875^2}{0.1^2} \times \left(\frac{237.07 - 207.44}{0.13 - 0.12}\right)$$

Sensitiveness of Governor :-

A governor is said to be sensitive when it readily responds to a small change of speed. The movement of the sleeve for a fractional change of speed is the measure of sensitivity.

Mathematically, sensitiveness $= \dfrac{\text{Mean speed}}{\text{Range of speed}}$

$$= \frac{N}{(N_2 - N_1)}$$

where N = mean speed of governor $= \dfrac{N_1 + N_2}{2}$

N_1 = minm speed

N_2 = maxm speed

so, $\boxed{\text{sensitiveness} = \dfrac{N_1 + N_2}{2(N_2 - N_1)}}$

Hunting :-

- A governor is said to be hunting if the speed fluctuates continously above and below the mean speed.

Isochronism :-

A governor with a range of speed zero, is known as an isochronous governor. For an isochronous governor.

$$\text{sensitiveness} = \frac{\text{Mean speed}}{\text{Range of speed}} = \infty$$

This means for all positions of the sleeve and ball, the governor has same speed.

- An isochronous governor is not practical due to friction at the sleeve.

For a Porter governor, we have

$$h_1 = \frac{g}{\omega_1^2} + \frac{Mg \pm f}{2m\omega_1^2}(1+K)$$

$$h_2 = \frac{g}{\omega_2^2} + \frac{Mg \pm f}{2m\omega_2^2}(1+K)$$

for equal arm lengths of the governors and intersecting at the spindle axis and neglecting frictional force.

$$h_1 = \frac{g}{\omega_1^2}\left(1+\frac{M}{m}\right) \quad h_2 = \frac{g}{\omega_2^2}\left(1+\frac{M}{m}\right)$$

for isochronism $\omega_1 = \omega_2$ i.e., $h_1 = h_2$

In case of Hartnell governor, neglecting friction

at ω_1,

$$m r_1 \omega_1^2 a = \frac{1}{2}(Mg + f s_1) b$$

at ω_2, $m r_2 \omega_2^2 a = \frac{1}{2}(Mg + f s_2) b$

for isochronism $\omega_1 = \omega_2$

$$\frac{m r_1 \omega^2}{m r_2 \omega^2} = \frac{Mg + f s_1}{Mg + f s_2}$$

$$\Rightarrow \boxed{\frac{r_1}{r_2} = \frac{Mg + f s_1}{Mg + f s_2}}$$

Stability :- A governor is said to be stable if it brings the speed of the engine to the required value without much hunting. The balls of the governor occupy a definite position for each speed of the engine within working range.

For max^m speed (safe), the condition

$$R_w = \frac{P}{2} + \frac{Q}{2}$$

$\Rightarrow 5395.5 = (0.167 + 5.041)\, v^2$

$\Rightarrow v^2 = 1036.002$

$\Rightarrow v = 32.18\ m/s.$

$= \frac{32.18 \times 3600}{1000} = \boxed{115.87\ km/hr.}$

Q.2 A racing car weighs 20 kN. It has a wheel base of 2m, track width 1 m and ht. of CG 0.3 m above the ground level. The engine flywheel rotates at 3000 rpm clockwise when viewed from the front. The moment of inertia of flywheel is 4 kg m² and mt. of each wheel is 3 kg m². Find the reactions betn the wheels and the ground when the car takes a curve of radius 15 m towards right at 30 km/hr, taking into account the gyroscopic and centrifugal effect. wheel radius is 0.4 m.

$$m r \omega_1^2 \times a = \left(\frac{Mg + f_s + f}{2}\right) b$$

$$m \times 0.075 \times (37.7 \times 1.01)^2 \times a = \left(\frac{5 \times 9.81 + f_s + 35}{2}\right)$$

$$m \times .075 \times (37.7 \times .099)^2 = \left(\frac{5 \times 9.81 + f_s - 35}{2}\right)$$

$\boxed{m = 8.21\ kg.}$